1. The system function of a discrete-time system is:

\[ H(z) = \frac{2}{1 - e^{-0.2z^{-1}}} - \frac{1}{1 - e^{-0.4z^{-1}}} \]

a) Assume that this discrete-time filter was designed by the impulse invariance method with \( T_d = 1 \); i.e., \( h[n] = h_c(n) \), where \( h_c(t) \) is real. Find the system function \( H_c(s) \) of a continuous-time filter that could have been the basis for the design.

b) Assume that \( H(z) \) was obtained by the bilinear transform method with \( T_d = 1 \). Find the system function \( H_c(s) \) that could have been the basis for the design.

2. Consider a causal continuous-time system with impulse response \( h_c(t) \) and system function:

\[ H_c(s) = \frac{s + a}{(s + a)^2 + b^2} \]

a) Use impulse invariance to determine \( H_1(z) \) for a discrete-time system such that \( h_1[n] = Th_c(nT) \).

b) Plot the magnitude and frequency response for both the continuous-time and discrete-time filters using MATLAB assuming \( a = 1 \), \( b = 1 \) and \( T = 1 \). (Hint: Use MATLAB commands `freqs` and `freqz` to get the frequency responses).

3. A discrete-time low-pass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function:

\[ |H_c(j\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}} \]

The specifications for the discrete-time system are:

\[ 0.9 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.2\pi \]

\[ |H(e^{j\omega})| \leq 0.1, \quad 0.3\pi \leq |\omega| \leq \pi \]

Assume that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the desired discrete-time filter.

a) Sketch the tolerance bounds on the magnitude of the frequency response, \( |H_c(j\Omega)| \) of the continuous-time Butterworth filter such that after application of the impulse invariance method (i.e., \( h[n] = Th_c(nT_d) \)), the resulting discrete-time filter will satisfy the given design specifications. Assume that \( T_d = 1 \).
b) Determine the integer order N and the quantity $\Omega_c$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.

c) Determine system function $H(s)$ and get $H(z)$ by impulse invariance. (To simplify the calculation, zp2tf function in matlab can be used for additive form transfer function. Residue function can be used to do transform between partial fraction expansion and additive form of transfer function)

d) Use MATLAB to plot the magnitude and phase response of $H(z)$. (freqz can be used)

4. Determine the system function $H(z)$ of the lowest-order Chebyshev Type I digital filter that meets the following specifications: (Use $T_d = 1$ if impulse invariance method is chosen. Similarly, zp2tf and residue can be used in matlab)

   a) 1-dB ripple in the passband $0 \leq |\omega| \leq 0.2\pi$.

   b) At least 15 dB attenuation in the stopband $0.4\pi \leq |\omega| \leq \pi$.

   c) Intuitively, what happens to N (order of the filter), if the minimum stop band attenuation required is 30 dB instead of 15 dB as in part (b)?