1. Suppose that we wish to design an FIR lowpass filter with the following specifications:

\[ 0.92 < H(e^{j\omega}) < 1.02 \quad 0 \leq |\omega| \leq 0.63\pi, \]

\[ |H(e^{j\omega})| < 0.1 \quad 0.65\pi \leq |\omega| \leq \pi \]

by applying a window to the impulse response \( h_0[n] \) for the ideal discrete-time lowpass filter with cutoff \( \omega_c = 0.64\pi \).

a) For each the following windows: Hamming, Hanning, and Bartlett modify the MATLAB code 'HW8-prob1_code.m' to determine the minimum value of \( M \) that satisfies the specification.

b) To support your answer, for each window plot the frequency response of the filter you generated in part (a). Show that with \( M-1 \) the constraints are not satisfied.

2. An ideal discrete-time Hilbert transformer is a system that introduces -90° (-\( \pi/2 \) radians) of phase shift for 0 and +90° (\( \pi/2 \) radians) of phase shift for \(-\pi < \omega < 0\). The magnitude of the frequency response is constant (unity) for \(-\pi < \omega < 0\) and for \(0 < \omega < \pi\). Such systems are also called ideal 90° phase shifters.

\[ H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0. \end{cases} \]

a) Plot the phase response of this system for \(-\pi < \omega < \pi\)

b) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use \( H(e^{j\omega}) \) given above, to determine the ideal impulse response \( h[n] \) if the FIR system is to be such that \( h[n]=0 \) for \( n < 0 \) and \( n > M \).

c) What type(s) of FIR linear-phase systems (I, II, III, or IV) can be used to approximate the ideal Hilbert transformer in part (a)?

3. Consider designing a discrete-time filter with system function \( H(z) \) from a continuous-time filter with rational system function \( H_c(s) \) by the transformation.

\[ H(z) = H_c\left(s\right)|_{s=\frac{\beta}{1-\alpha(z^{-\alpha})}} \]

Where \( \alpha \) is a nonzero integer and \( \beta \) is real.

a) If \( \alpha > 0 \), for what values of \( \beta \) does a stable, causal continuous-time filter with rational \( H_c(s) \) always lead to a stable, causal discrete-time filter with rational \( H(z) \)?

b) If \( \alpha < 0 \), for what values of \( \beta \) does a stable, causal continuous-time filter with rational \( H_c(s) \) always lead to a stable, causal discrete-time filter with rational \( H(z) \)?
4. Download the two attached files. A piece of music is added with a high-pass noise. Please design a low-pass filter to eliminate this noise. The specification of that high-pass noise is:

\[
\begin{align*}
    f_{\text{stop}} &= 10 \text{ kHz} \\
    f_{\text{pass}} &= 12 \text{ kHz}
\end{align*}
\]

The original music has a sample rate equals to \( f_{\text{sample}} = 44.1 \text{ kHz} \).

You can use command ‘sound’ in MATLAB to play the music. \([\text{sound}(y, f_s)\) sends audio signal \( y \) to the speaker at sample rate \( f_s \)].

Choose one of the following filter types to design the filter in MATLAB:

- Chebyshev type I
- Chebyshev type II
- Butterworth

a) Fill out the attached matlab code with calculated design variables and include generated plots.

b) Bonus: Solve again using a second filter type.