Lecture 13: MOS Capacitors – Electrostatics and Capacitance

Sze And Ng: Chapter 4

Announcements

Homework 3/4:
• Online now.
• Due Today. Please bring it to me if you have not already done so.
• I will return it and post solutions on 5th March.

Homework 4/4:
• Online Tuesday 5th March.
• Due Tuesday 12th March at the start of the lecture (08:30am).
• I will return it on the day of the final (18th March).
• I will post the solutions when I return your homeworks.
• Homework 4 will consist of content covered in Lectures 12-15.
Last Time

- We just looked qualitatively at band diagrams for MOS capacitors, and explained how we expect them to behave under an applied bias.

Lecture 13

- Charge Density.
- Electrostatics.
- Capacitance Voltage Analysis.
Charge Density

• We can infer from our study of Schottky contacts that most of the interesting behavior occurs in the semiconductor.
  • Mobile charge distributions in metals or insulators are likely to be either $\delta$-functions or zero.
  • In a semiconductor we say that mobile charge generally is due to:

$$\rho(x) = e[p - n + N_D^+ - N_A^-]$$  \hspace{1cm} (1)

• $N_D^+$ is ionized donor density.
• $N_A^-$ is ionized acceptor density.
• $n$ is electron density. \hspace{0.5cm} • $p$ is hole density.
**Charge Density**
- For our p-type semiconductor we say specifically:
  \[ \rho(x) = e\left[p_p - n_p + N_D^+ - N_A^-\right] \]  
- Where the subscripts denote the semiconductor type.
- Recall from last time we expect the bands to look like:
  - Bulk (p-type) potential:
    \[ \phi_{Bp} = \frac{E_i - E_{FS}}{e}\bigg|_{\text{bulk}} \]  
  - Potential at x:
    \[ \phi(x) = -\frac{E_i(x) - E_i|_{\text{bulk}}}{e} \]  

![Diagram showing charge density and potential](image)

**Carrier Concentration**
- We also showed we were able to describe the charge density in terms of equilibrium charge density:
  - Hole concentration at x:
    \[ p_p(x) = p_{p0} \exp\left(-\frac{e\phi(x)}{k_B T}\right) \]  
  - Hole concentration in equilibrium:
    \[ p_{p0} = n_{p0} \exp\left(\frac{e\phi(x)}{k_B T}\right) \]  
  - Where:
    - \( p_p \) hole concentration at x.
    - \( p_{p0} \) hole concentration in equilibrium
    - \( n_p \) hole concentration at x.
    - \( n_{p0} \) hole concentration in equilibrium
**Charge Density**

- We can put (7) and (8) into our equation for charge density:

\[
\rho(x) = e \left[ p_p - n_p + N_D^+ - N_A^- \right] \tag{2}
\]

\[
\rho(x) = e \left[ p_p \exp \left( -\frac{e\phi(x)}{k_B T} \right) - n_p \exp \left( \frac{e\phi(x)}{k_B T} \right) + N_D^+ - N_A^- \right] \tag{7}
\]

- We are also going to say that a long way from the interface, each impurity provides one charge:

\[
N_D^+ \approx n_p \quad (8) \quad N_A^- \approx p_p \quad (9)
\]

\[
\rho(x) = e \left[ p_p \exp \left( -\frac{e\phi(x)}{k_B T} \right) - n_p \exp \left( \frac{e\phi(x)}{k_B T} \right) + n_p - p_p \right] \tag{10}
\]

**Electrostatic Potential**

- Putting (10) into Poisson’s Equation:

\[
\frac{d^2 \phi}{dx^2} = -\frac{e}{\epsilon_r \epsilon_0} \left[ p_p \exp \left( -\frac{e\phi}{k_B T} \right) - n_p \exp \left( \frac{e\phi}{k_B T} \right) + n_p - p_p \right] \tag{11}
\]

\[
\frac{d^2 \phi}{dx^2} = -\frac{e}{\epsilon_r \epsilon_0} \left[ p_p \left[ \exp \left( -\frac{e\phi}{k_B T} \right) - 1 \right] - n_p \left[ \exp \left( \frac{e\phi}{k_B T} \right) - 1 \right] \right] \tag{12}
\]

- To make progress, we now have to integrate with respect to the potential.
  - **We are not** integrating with respect to space!
  - This is counter-intuitive, but it will become clear why we do this shortly.
Electric Field

• Integrate w.r.t. \( \phi \):
\[
\int_\gamma^\gamma \frac{d^2\phi}{dx^2} d\phi = -\frac{e}{\epsilon_r \epsilon_0} \int_\gamma^\gamma \left[ p_{p0} \left[ \exp \left( \frac{-e\phi}{k_B T} \right) - 1 \right] - n_{p0} \left[ \exp \left( \frac{e\phi}{k_B T} \right) - 1 \right] \right] d\phi
\]

• How do we decide limits?
  • Right now we are using a mathematical trick, rather than explicitly encapsulating something from our description / thought experiment.
  • So we will just integrate from 0 to our \( \phi \) of interest.
\[
\int_0^\phi \frac{d^2\phi}{dx^2} d\phi = -\frac{e}{\epsilon_r \epsilon_0} \int_0^\phi \left[ p_{p0} \left[ \exp \left( \frac{-e\phi}{k_B T} \right) - 1 \right] - n_{p0} \left[ \exp \left( \frac{e\phi}{k_B T} \right) - 1 \right] \right] d\phi
\]

(13)

• We can show that the left hand side (LHS) is:
\[
\text{LHS} = \int_0^\phi \frac{d^2\phi}{dx^2} d\phi = \frac{1}{2} \mathcal{E}^2
\]

(14)

• \( \mathcal{E} \) is the electric field strength.
• The right hand side (RHS) is a lot easier:
\[
\text{RHS} = \frac{k_B T}{\epsilon_r \epsilon_0} \left[ \left( \exp \left( \frac{-e\phi}{k_B T} \right) + \frac{e\phi}{k_B T} - 1 \right) + \frac{n_{p0}}{p_{p0}} \left( \exp \left( \frac{e\phi}{k_B T} \right) - \frac{e\phi}{k_B T} - 1 \right) \right]
\]

(15)
Electric Field

- Equating (14) and (15):
  \[ \frac{1}{2} \varepsilon^2 = \frac{k_B T p_0}{\varepsilon_r \varepsilon_0} \left[ \left( \exp \left( -\frac{e\phi}{k_B T} \right) + \frac{e\phi}{k_B T} - 1 \right) + \frac{n_{p0}}{p_{p0}} \left( \exp \left( \frac{e\phi}{k_B T} \right) - \frac{e\phi}{k_B T} - 1 \right) \right] \] (16)

- Re-arranging in terms of the electric field:
  \[ \varepsilon = \sqrt{\frac{k_B T p_0}{\varepsilon_r \varepsilon_0}} \left[ \left( \exp \left( -\frac{e\phi}{k_B T} \right) + \frac{e\phi}{k_B T} - 1 \right) + \frac{n_{p0}}{p_{p0}} \left( \exp \left( \frac{e\phi}{k_B T} \right) - \frac{e\phi}{k_B T} - 1 \right) \right] \] (17)

- Introduce the extrinsic Debye length for holes:
  \[ L_D = \sqrt{\frac{k_B T \varepsilon_r \varepsilon_0}{p_{p0} \varepsilon^2}} \] (18)
  \[ \frac{1}{L_D} = \sqrt{\frac{p_{p0} e^2}{k_B T \varepsilon_r \varepsilon_0}} \] (19)

Electric Field

- To make things slightly more tidy, we define \( F \):
  \[ F \left( \frac{e\phi}{k_B T}, \frac{n_{p0}}{p_{p0}} \right) = \left[ \left( \exp \left( -\frac{e\phi}{k_B T} \right) + \frac{e\phi}{k_B T} - 1 \right) + \frac{n_{p0}}{p_{p0}} \left( \exp \left( \frac{e\phi}{k_B T} \right) - \frac{e\phi}{k_B T} - 1 \right) \right]^{1/2} \] (20)

- This allows us to write the electric field a little bit more compactly:
  \[ \varepsilon(x) = \pm \frac{\sqrt{2k_B T}}{eL_D} F \left( \frac{e\phi}{k_B T}, \frac{n_{p0}}{p_{p0}} \right) \] (21)

- Where:
  \[ \pm = \begin{cases} + & \text{for } \phi > 0 \\ - & \text{for } \phi < 0 \end{cases} \] (22)
  \[ L_D = \frac{\sqrt{k_B T \varepsilon_r \varepsilon_0}}{\sqrt{p_{p0} e^2}} \] (18)
Charge Density

- It turns out we get most of the important information through the 2-dimensional charge density $Q_{SC}$.
- We use $Q_{SC}$ in the same context as with Schottky diodes: 2-dimensional charge density in semiconductor depletion region. Currently unknown.
- E.g. $Q_{SC}$ is C/cm$^2$ in the semiconductor.
- From charge balance, we get charge in the metal:

  \[ Q_M = -Q_{SC} \quad (23) \]

Charge Density

- We can use Gauss’ Law to express 2-dimensional charge density in terms of electric field strength:

  \[ Q_{SC} = -\varepsilon_r \varepsilon_0 \varepsilon = \mp \sqrt{2} \varepsilon_r \varepsilon_0 k_B T \frac{e\phi}{eL_D} F \left( \frac{e\phi}{k_B T}, \frac{n_p}{p_p} \right) \quad (24) \]

- The behaviour is encapsulated in $F$.
  - We are just going to evaluate it under specific conditions (depletion, accumulation, etc.).
  - But first, we are going to make one last charge to $F$.
  - The law of mass-action states:

  \[ n_i^2 = p_p n_p \quad (25) \]
Charge Density

\[ n_i^2 = p_0 n_0 \]  \hspace{1cm} n_p = \frac{n_i}{p_0} \]  \hspace{1cm} (26)

- We can use this to say:

\[ \frac{n_p}{p_0} = \left( \frac{n_i}{p_0} \right)^2 \]  \hspace{1cm} (27)

- From Boltzmann Statistics, we also say:

\[ p_0 = n_i \exp \left( - \frac{[E_i - E_F]}{k_B T} \right) \]  \hspace{1cm} (28)

- From our diagram we can also deduce:

\[ p_0 = n_i \exp \left( \frac{\phi_{BP}}{k_B T} \right) \]  \hspace{1cm} (29)

Combining (27) and (29) we say:

\[ \frac{n_p}{p_0} = \exp \left( - \frac{2\phi_{BP}}{k_B T} \right) \]  \hspace{1cm} (30)

- Which we can put into our \( F \) function:

\[
F \left( \frac{e \phi}{k_B T}, \frac{n_p}{p_0} \right) = \left[ \left( \exp \left( - \frac{e \phi}{k_B T} \right) + \frac{e \phi}{k_B T} - 1 \right) + \frac{n_p}{p_0} \left( \exp \left( \frac{e \phi}{k_B T} \right) - \frac{e \phi}{k_B T} - 1 \right) \right]^{1/2}
\]  \hspace{1cm} (20)

\[
F = \left[ \left( \exp \left( - \frac{e \phi}{k_B T} \right) + \frac{e \phi}{k_B T} - 1 \right) + \exp \left( - \frac{e \phi - 2 \phi_{BP}}{k_B T} \right) + \exp \left( - \frac{2e \phi_{BP}}{k_B T} \right) \left[ - \frac{e \phi}{k_B T} - 1 \right] \right]^{1/2}
\]  \hspace{1cm} (31)
Surface Potential

- To understand the behaviour of the capacitor, let us first consider the potential at the surface.
- I.e. consider \( \phi(x = 0) = \phi_S \):

\[
Q_{sc} = \mp \sqrt{2\varepsilon_r\varepsilon_0 k_B T} \frac{e\phi_S}{eL_D} F\left(\frac{e\phi_S}{k_BT}, \frac{n_{p0}}{p_{p0}}\right)
\]

- Where:

\[
F\left(\frac{e\phi_S}{k_BT}, \frac{n_{p0}}{p_{p0}}\right) = \sqrt{\left(\exp\left(-\frac{e\phi_S}{k_BT}\right) + \exp\left(\frac{e\phi_S}{k_BT}\right) - 1\right) + \exp\left(\frac{e\phi_S - 2\phi_{FP}}{k_BT}\right) + \exp\left(-\frac{2\phi_{FP}}{k_BT}\right) - 1}\]

- We will just evaluate \( F \) under a few different circumstances to infer behavior.

1. Flat Band

- Flat band is the situation where the surface potential is zero:

\[
\phi(x) = \phi_S
\]

- Note this is not necessarily zero bias. Depending on the doping density and relative Fermi Energies, we may have to apply a bias to flatten out the bands.
1. Flat Band

• Let’s evaluate the $F$ function for flat band conditions:

$$F \left( \frac{e\phi_s}{k_BT}, \frac{n_{p0}}{p_{p0}} \right) = \sqrt{\left( \exp \left( \frac{e\phi_s}{k_BT} \right) + 1 \right) + \exp \left( -\frac{2e\phi_{BB}}{k_BT} \right)} + \exp \left( -\frac{2e\phi_{BB}}{k_BT} \right) - 1}$$

• If $\phi_s = 0$:

$$F \left( 0, \frac{n_{p0}}{p_{p0}} \right) = 1 + \exp \left( -\frac{2e\phi_{BB}}{k_BT} \right) - \exp \left( -\frac{2e\phi_{BB}}{k_BT} \right) - 1}$$

• Hence we can also say for flatband:

$$\mathcal{E} = Q_{sc} = 0$$

---

1. Flat Band

• So unlike our pn-junction or Schottky contact, under flat-band conditions the donor/acceptor atoms in the semiconductor in the MOS capacitor are not ionized.
  • The dopant atoms remain neutral.
  • No charge-transfer / annihilation takes place.
2. Accumulation

- So what happens when the surface potential is negative ($\phi_s < 0$)?
- Recall:

$$\phi(x) = -\frac{E_i(x) - E_i|_{\text{bulk}}}{e} \quad (4)$$

- So for a p-type semiconductor, we define upward band bending as a negative $\phi_s$.

$$F \left( \frac{e\phi_s}{k_BT} \frac{n_0}{p_0} \right) = \sqrt{\left[ \exp \left( -\frac{e\phi_s}{k_BT} \right) + \exp \left( \frac{e\phi_s}{k_BT} - 1 \right) \right] + \exp \left( \frac{e|\phi_s - 2\phi_B|}{k_BT} \right) + \exp \left( -\frac{2e\phi_B}{k_BT} \right) - \frac{e\phi_s}{k_BT} - 1} \quad (33)$$

- Which terms dominate if $\phi_s < 0$?

The most dominant terms are clearly going to be those where we have an exponential to some positive number.

- So we simply deduce the proportionality relationship:

$$Q_{sc} \propto \exp \left( -\frac{e\phi_s}{2k_BT} \right) \propto \exp \left( \frac{e|\phi_s|}{2k_BT} \right) \quad (36)$$

- So in accumulation we say the 2-dimensional charge density is exponentially dependent on surface potential.
3. Depletion & Weak Inversion

- We are going to combine these together.
- Recall from last time that they occur under similar band bending conditions:

\[ V = \frac{e\phi_s}{k_BT} + \frac{e\phi_{np}}{k_BT} - 1 + \exp\left(\frac{e\phi_s - 2\phi_{BP}}{k_BT}\right) + \exp\left(-\frac{2e\phi_{BP}}{k_BT}\right) - \frac{e\phi_s}{k_BT} - 1 \]

- Since \( \phi_s \) is positive, we can neglect the first term.
- What about the middle exponential?
- \( \phi_{BP} \) is positive.
- \( \phi_s < 2\phi_{BP} \).
3. Depletion & Weak Inversion

\[ F \left( \frac{e\phi_s}{k_B T}, \frac{n_p}{p_n} \right) = \sqrt{\left( \exp \left( \frac{e\phi_s}{k_B T} \right) + \exp \left( \frac{e\phi_s}{k_B T} - 1 \right) \right) + \exp \left( \frac{e|\phi_s| - 2\phi_{BP}}{k_B T} \right) + \exp \left( - \frac{2e\phi_{BP}}{k_B T} \right) + \frac{e\phi_s}{k_B T} - 1} \]

- So we are left with the terms which are linear (inside the square root).
- We know that \( \phi_{BP} \) is positive, so:
  \[ \exp \left( - \frac{2e\phi_{BP}}{k_B T} \right) < 1 \]
- Hence in this case the dominant regime term is:

\[ Q_{sc} \propto \frac{e\phi_s}{\sqrt{k_B T}} \]

\[ Q_{sc} \propto \sqrt{\phi_s} \quad (37) \]

4. Strong Inversion

- Finally we have the strong inversion case.
  - Recall, we define this as the situation when: \( \phi_s > 2\phi_{BP} \)
  - And both are positive.

- Recall, this is distinct from weak inversion in that:
  - \( E_C - E_{FS} \) at surface < than \( E_{FS} - E_V \) in bulk.
  - \( p_s < p_B < n_s \).
- More minority carriers the majority carriers in bulk.
4. Strong Inversion

\[ F \left( \frac{e\phi_s}{k_B T}, \frac{n_D}{p_D} \right) = \left( \exp \left( -\frac{e\phi_s}{k_B T} \right) + \exp \left( \frac{e\phi_s - 2\phi_B}{k_B T} \right) \right) + \exp \left( -\frac{2e\phi_B}{k_B T} \right) \left[ -\frac{e\phi_s}{k_B T} - 1 \right] \]

- If \( \phi_s \) is positive and \( \phi_s > 2\phi_B \), when we can identify from above that the most dominant term will now be the positive exponential.
- I.e.:
  \[ Q_{sc} \propto \exp \left( \frac{e[\phi_s - 2\phi_B]}{2k_B T} \right) \]  
  (38)

Charge Density

- We can put this all together to describe the different operating regimes.
- 1. Flatband \( \phi_s = 0 \):
  \[ Q_{sc} = 0 \]  
  (35)
- 2. Accumulation \( \phi_s < 0 \):
  \[ Q_{sc} \propto \exp \left( \frac{e|\phi_s|}{2k_B T} \right) \]  
  (36)
- 3. Depletion and weak inversion \( \phi_s < 2\phi_B \):
  \[ Q_{sc} \propto \sqrt{|\phi_s|} \]  
  (37)
- 4. Strong inversion \( \phi_s > 2\phi_B \):
  \[ Q_{sc} \propto \exp \left( \frac{e[\phi_s - 2\phi_B]}{2k_B T} \right) \]  
  (38)
Charge Density

- The textbook plots this behavior explicitly:

Electrostatics
Electrostatic Potential

• In order to derive the band-diagram (quantitatively) we need to evaluate the electrostatic potential.

• Energy bands can just then be derived from potential (+ a constant) via:

\[ \phi = -\frac{E_i}{e} \]

(39)

• Where:
  • \( \phi \) is the electrostatic potential.
  • \( E_i \) is the intrinsic energy.
  • \( e \) is the fundamental unit of charge.

Electrostatic Potential

• We can use Poisson’s Equation (in one dimension) to get the electrostatic potential from charge density:

\[ \frac{d^2 \phi}{dx^2} = -\frac{\rho(x)}{\epsilon_r \epsilon_0} \]

(40)

• Where:
  • \( \phi \) is the electrostatic potential.
  • \( x \) is the direction of interest.
  • \( \rho \) is the charge density.
  • \( \epsilon_r \) is the relative permittivity.
  • \( \epsilon_0 \) is the relative permittivity.
Electrostatic Potential

- $n$-type
- $p$-type
- Metal
- $n$-type

\[ \rho \cdot N_D \cdot \varepsilon \cdot W_{dn} \]

MOS Capacitors

- For our metal-oxide(insulator)-semiconductor structure, we have 3 different layers.

- This is under flat-band conditions.
Charge Density

• So we have seen that the charge density distribution depends on band bending.

1). Flat-band:
• Under flat-band conditions we expect no mobile charge density and no depletion region:

\[ Q_{sc} = -Q_M \]

2). Accumulation
• Accumulation means holes accumulate in the p-type semiconductor.
• From charge balance, we get charge in the metal:

\[ Q_M = -Q_{sc} \]
**Charge Density**

3a). Depletion

- We have repelled some holes away from their host atoms. Hence we have a depletion region.
- We get charge density in the metal the same way:

\[ Q_M = -Q_{sc} \]  

(23)

3b). Weak Inversion

- Now we also have some electrons (more negative charge) appear at the interface.
- We get charge density in the metal the same way:

\[ Q_M = -Q_{sc} \]  

(23)
Charge Density

4). Strong Inversion

- Finally we have the strong inversion case.
- More electrons at the interface.

\[ \rho_p \]

4). Strong Inversion

\[ E_c \]

\[ d \]

\[ e \phi_x \]

\[ e \phi_{bb} \]

\[ E_{E_m} \]

\[ E_{ps} \]

\[ E_i \]

\[ W \]

\[ Q_N \]

\[ Q_N \]

\[ n_x N_A \]

\[ \text{Metal} \]

\[ \text{Oxide} \]

\[ \text{p-type} \]

Electric Field

- Evaluating the electric field explicitly is pretty complicated here.

\[ \mathcal{E}(x) = \pm \frac{\sqrt{2} k_B T}{e L_D} F \left( \frac{e \phi}{k_B T}, \frac{n_{p0}}{p_{p0}} \right) \quad (21) \]

- Where:

\[ \pm = \begin{cases} + & \text{for } \phi > 0 \\ - & \text{for } \phi < 0 \end{cases} \quad (22) \]

\[ L_D = \sqrt{\frac{k_B T e_F e_0}{p_{p0} e^2}} \quad (18) \]

\[ F \left( \frac{e \phi}{k_B T}, \frac{n_{p0}}{p_{p0}} \right) = \left[ \left( \exp \left( \frac{-e \phi}{k_B T} + \frac{e \phi}{k_B T} - 1 \right) + \frac{n_{p0}}{p_{p0}} \left( \exp \left( \frac{e \phi}{k_B T} \right) - \frac{e \phi}{k_B T} - 1 \right) \right) \right]^{1/2} \quad (20) \]

- So we will not go through it in detail here.
Textbook

- Sze and Ng give an example for the case for strong inversion.
- They divide the voltage drop up (as we did last lecture):
  \[ V = V_i + \phi_s \] (41)
- \(V_i\) is the voltage dropped across the insulator.
- \(\phi_s\) is the voltage dropped across the semiconductor.

Capacitance-Voltage (CV) Analysis
Capacitance

- We describe the total capacitance ($C$) across the capacitor as two capacitors in series.
  
  - We can hence say:
    
    $$\frac{1}{C} = \frac{1}{C_i} + \frac{1}{C_D}$$  \hspace{1cm} (42)

  - The charge in the metal and the semiconductor equal.

  - Hence we can also say:
    
    $$C = \frac{dQ_{sc}}{dV} = \frac{dQ_M}{dV}$$  \hspace{1cm} (43)

Capacitance

- We have an equation (albeit a reasonably complicated one) for charge density at the surface of the MOS capacitor:
  
  $$Q_{sc} = \mp \frac{\sqrt{2}\varepsilon \varepsilon_0 k_B T}{e L_D} F\left(\frac{e \phi_s}{k_B T}, \frac{n_{p0}}{p_{p0}}\right)$$  \hspace{1cm} (32)

  - We can therefore calculate capacitance across the depletion region:
    
    $$C_D = \frac{dQ_{sc}}{d\phi_s} = \frac{\varepsilon \varepsilon_0}{\sqrt{2} L_D} \left(1 - \exp\left(-\frac{e \phi_s}{k_B T}\right) + \frac{n_{p0}}{p_{p0}} \left(\exp\left(\frac{e \phi_s}{k_B T}\right) - 1\right)\right)$$  \hspace{1cm} (44)
Charge Density

- Note that we re-define the charge density in the semiconductor depending on the situation.
- We just say:
  - \( Q_{sc} = Q_A \) in accumulation mode.
  - \( Q_{sc} = Q_n + Q_D \) in depletion or inversion.
- Where:
  - \( Q_n \) is the inversion charge per unit area.
  - \( Q_D \) is the depletion region charge per unit area:

\[
Q_D = eN_A W \quad (45)
\]
\[
W = \sqrt{\frac{2\varepsilon_0 \varepsilon_r \phi_s}{eN_A}} \quad (46)
\]

CV Analysis

- The C-V technique is the most convenient method to explore the device physics of MOS interfaces.
- We saw last time that the applied voltage is dropped partly across the insulator and partly across the semiconductor:
CV Analysis

- One of the electrostatic boundary conditions specifies that the normal component of the electric displacement is discontinuous across the interface by the amount of interface charge per unit area.

- If we assume (ideally) that there is no charge present at the interface, this electrostatic boundary condition results in:

\[ \varepsilon_{r1}\varepsilon_0E_i = \varepsilon_{rS}\varepsilon_0E_s \]  

(47)

- \( \varepsilon_{r1}, \varepsilon_{rS} \): relative permittivity of the insulator, semiconductor, respectively.

- \( E_i, E_s \): electric field across the insulator, semiconductor, respectively.

CV Analysis

- From Gauss’ law, we obtain an expression for the electric field at the semiconductor/insulator interface:

\[ E_s = \frac{Q_{Sc}}{\varepsilon_{rS}\varepsilon_0} \]  

(48)

- Note: Eqn. (38) is equal to Eqn. (24) except that we are not worrying about the sign. Taking the sign into consideration is a real headache and adds nothing of importance in the analysis since signs just account for which direction the electric field points and the polarity of the charge giving rise to the electric field.
CV Analysis

• If we also assume that there is no charge present in the oxide (another ideal assumption), we can write an expression for the electric field in the insulator as

\[ \mathcal{E}_i = \frac{V_i}{d} \]  
(49)

• \( d \) is the insulator thickness.

• Substitution of Eqns (48) and (49) into Eqn. (47) gives:

\[ V_i = \frac{Q_{sc}d}{\varepsilon_{ri}\varepsilon_0} = \frac{Q_{sc}}{C_i} \]  
(50)

• \( C_i \) is the insulator capacitance / area.

CV Analysis

• Substituting (50) into (41):

\[ V = \frac{Q_{sc}}{C_i} + \phi_s \]  
(51)

• Take the first derivative of (51) with respect to voltage:

\[ \frac{dV}{dV} = 1 = \frac{1}{C_i} \frac{dQ_{sc}}{dV} + \frac{d\phi_s}{dV} \]  
(52)

• Important: for now we assume this is the small-signal capacitance. I.e.:

\[ C_i \approx C_i(V) \]  
(53)
CV Analysis

• Express the last term in (52) using the chain rule:

\[
\frac{d\phi_s}{dV} = \frac{d\phi_s}{dQ_{sc}} \frac{dQ_{sc}}{dV} \quad (54)
\]

• Substitute into Equation (52):

\[
1 = \frac{1}{C_i} \frac{dQ_{sc}}{dV} + \frac{d\phi_s}{dQ_{sc}} \frac{dQ_{sc}}{dV}
\]

\[
1 = \frac{dQ_{sc}}{dV} \left( \frac{1}{C_i} + \frac{d\phi_s}{dQ_{sc}} \right) \quad (55)
\]

• Identify the capacitance across the depletion region:

\[
C_D = \frac{d\phi_s}{dQ_{sc}} \quad (44)
\]

CV Analysis

• Re-arranging (50):

\[
Q_{sc} = V_i C_i \quad (56)
\]

• Differentiating with respect to voltage (once again assuming small-signal):

\[
\frac{dQ_{sc}}{dV} = C_i \frac{dV_i}{dV} = C \quad (57)
\]

• Differentiating (41) with respect to voltage:

\[
\frac{dV}{dV} = 1 = \frac{dV_i}{dV} + \frac{d\phi_s}{dV} \quad \rightarrow \quad \frac{dV_i}{dV} = 1 - \frac{d\phi_s}{dV} \quad (58)
\]
Berglund Integral

• Substituting (58) into (57):

\[
C_i \left(1 - \frac{d\phi_s}{dV}\right) = C \quad \rightarrow \quad \frac{d\phi_s}{dV} = 1 - \frac{C}{C_i} \quad (59)
\]

• Split up the differentials:

\[
\Delta \phi_s = \int_{V_1}^{V_2} \left(1 - \frac{C(V)}{C_i}\right) dV \quad (60)
\]

• Evaluation of this integral allows us to find the change (modulation) in the surface potential \(\phi_s\), for a given applied voltage interval.

• This is called the Berglund Integral.

Small Signal Capacitance

• We have ignored one important point up to now: the measured capacitance is a small-signal, differential (dynamic, AC) capacitance.

• We can describe the CV measurement as below (from Lecture 3):

Sine wave, typically \(f = 1\) MHz, \(V_{AC} = 25\) mV (rms)

\(V_{DC} = 0\) - 10 V
Small Signal Capacitance

- In order for an AC voltage to yield an AC capacitance, the AC signal frequency must be low enough that equilibrium is maintained with regard to charge balance.
- The depletion layer charge equilibrium is easily maintained because the mechanism for establishment of equilibrium is the movement of majority carriers from the neutral region of the semiconductor to the edge of the space charge region. This occurs on the order of the dielectric relaxation time:

\[ \tau_{DR} = \rho \epsilon_0 \epsilon_{ri} \]  

Inversion

- On the other hand, the rate-limiting process for establishing equilibrium in inversion is usually minority carrier generation.
- Minority carrier generation can occur via:
  - Carrier generation and transport in the space charge region (related to \( n_i \)).
  - Carrier generation and transport in the neutral bulk (related to \( n_i \)).
  - Carrier generation by interface states.
- Typically, the time constant associated with the fastest type of generation process is \(~10\) ms (100 Hz).
**Summary**

- We quantified the electric field and the electric charge density:
  
  ![Diagram of electric field and charge density](image)

- We then evaluated the capacitance:

  ![Diagram of capacitance evaluation](image)

**Next Time**

- We will look at $CV$ curves.

  ![Diagram of $CV$ curve](image)

- And interface states: