Lecture 7: Equivalent Circuit, Boundary Conditions and Surface Recombination

Sze And Ng: Chapter 2

Announcements

Homework 2/4:

• Is online now.
• Due Thursday 31st January at the start of the lecture (08:30am).
• I will return it one week later (5th February).
• Homework 1 consists of content covered in Lectures 5 - 6.
Announcements

Midterm

- Thursday February 7th at 08:30am in STAG 211
- Exam will last 80 minutes.
  - The exam will start exactly at 08:30am!
- Closed book and closed notes.
- You can, and are expected to, use a calculator.
- Choose 2 out of 3 questions.
  - If you answer 3 I will take best 2 scores.
- It will contribute 30% of overall grade for class.
- The exam will material covered in lectures 2-8 (inclusive).
- There will be 25 marks per question. 50 total.

Last Time

- We looked at some of the ways in which the current voltage behavior of pn-junctions can differ from the ideal diode equation we derived last time.
  - We also spent a bit of time looking at what happens if we apply a very large reverse bias, and what causes breakdown.
Lecture 7

• Large-Signal Equivalent Circuit.
• Small-Signal Equivalent Circuit.
• Fletcher’s Boundary Conditions.
• Other Boundary Conditions.
• Surface Recombination

Large-Signal Equivalent Circuit
**Equivalent Circuit**

- Our goal is to formulate the equivalent circuit for a pn junction. This two-terminal device will be defined by the IV characteristics of a diode.

- I.e. we seek an equation for the total current flowing through the diode.
  - It turns out we need further modifications to \( I(V) \).

**Conduction Current**

- The current through a diode can be both a conduction current as well as a displacement current:

  \[
i_{\text{diode}} = i_{\text{cond}} + i_{\text{displace}}\tag{1}
\]

- Where:
  - \( i_{\text{diode}} \) is the total diode current.
  - \( i_{\text{cond}} \) is the total conduction current.
  - \( i_{\text{displace}} \) is the total displacement current.

  \[
i_{\text{cond}} = I_0 \left[ \exp \left( \frac{eV_D}{nk_BT} \right) - 1 \right]\tag{2}
\]
Displacement Current

• What about $i_{\text{displace}}$?
• The displacement current is due to capacitance:

$$i_{\text{displace}} = C_{\text{diode}} \frac{dV_D}{dt} \quad (2)$$

• $C_{\text{diode}}$ is the small-signal capacitance of the diode.
• Applying a bias across a capacitor will create a charge that must be supplied by displacement current.
• We can obtain an explicit expression for $C_{\text{diode}}$ if we assume a one sided step junction that operates at low frequency.

Diode Capacitance

• The capacitance of the diode is itself formed of two contributions:
• $C_{\text{depl}}$ is the capacitance due to the depletion of the majority carriers. I.e. due to the depletion region in the pn junction.

$$C_{\text{diode}} = C_{\text{depl}} + C_{\text{diff}} \quad (3)$$

• For now consider one-sided abrupt junctions.
Depletion Capacitance

- In Lecture 3 (Equation 34) we showed the depletion capacitance could be given by:

\[ C_{\text{depl}} = \frac{\varepsilon_r \varepsilon_0}{\sqrt{2L_D}} [(\beta \phi_{bi} \pm \beta V_D - 2)]^{-1/2} \]  

- "+" is for reverse bias and "-" is for forward bias.
- \( \varepsilon_r \): relative permittivity.
- \( \epsilon_0 \): vacuum permittivity.
- \( \phi_{bi} \): built-in potential.
- \( \beta = \frac{e}{k_B T} \)  
- \( L_D = \left[ \frac{\varepsilon_r \varepsilon_0 k_B T}{e^2 N_b} \right]^{1/2} \) 

Diffusion Capacitance

- Up until now, we have considered only capacitance due to space charge in the depletion layer (covered or uncovered due to movement of majority carriers).
- Under sufficiently large forward biases (or under breakdown conditions) we also must consider the additional charge of the minority carriers.
Diffusion Capacitance

- This leads to a so-called diffusion capacitance, $C_D$, which is a capacitance due to rearrangement of delocalized minority carriers.

- Sze derives the low frequency diffusion capacitance ($C_{D0}$):

\[
C_{\text{diff}} = \frac{q^2}{2k_BT} \left( L_p p_n + L_n n_p \right) \exp \left( \frac{eV_D}{k_BT} \right)
\]  

(7)

- $L_p$ is the hole minority carrier diffusion length.

- $L_n$ is the electron minority carrier diffusion length.

- $p_n$ is the hole minority carrier concentration.

- $n_p$ is the electron minority carrier concentration.

Depletion Capacitance

\[
C_{\text{depl}} = \frac{\epsilon_r \epsilon_0}{\sqrt{2}L_D} \left[ (\beta \phi_{bi} \pm \beta V_D - 2) \right]^{-1/2}
\]  

(4)

- Although we could use this expression to obtain a large signal equivalent circuit model for the pn junction, it is more convenient to express the depletion capacitances in a more general form:

\[
C_{\text{depl}} = C_{\text{depl}}(0) \left[ 1 - \frac{V_D}{\phi_{bi}} \right]^{-1/2}
\]  

(8)

- Where:

\[
C_{\text{depl}}(0) = \frac{\epsilon_r \epsilon_0}{W(0)}
\]  

(9)
Depletion Capacitance

- Only valid when $V_D \leq \phi_{bi}/2$:

![Graph showing depletion capacitance](image)

Diffusion Capacitance

$$C_{\text{diff}} = \frac{q^2}{2k_B T} \left( L_p p_{n0} + L_n p_{n0} \right) \exp \left( \frac{eV_D}{k_B T} \right)$$

(7)

- Next, we want to find an alternative expression for the diffusion capacitance, $C_{\text{diff}}$.
- We do this in a simple manner by first recognizing that charge is equal to current times time:

  $$q_{\text{diff}} = i_{\text{cond}} \times TT$$

(9)

- $TT$ is transit time, which is an ill-defined parameter which characterizes the time it takes to remove charge from the quasi-neutral region of the pn junction when the junction is turned off.
Transit Time

- Remember our carrier concentration diagram in **Forward bias**:

\[
\begin{array}{c|c|c}
\varepsilon = 0 & \varepsilon \neq 0 & \varepsilon = 0 \\
I & II & III \\
p_{po} \approx N_A & n_{po} \approx \frac{n_i^2}{N_A} & n_{no} \approx N_D \\
n_p & p_n & p_{no} \approx \frac{n_i^2}{N_D} \\
\end{array}
\]

- Remember our carrier concentration diagram in **Reverse bias**:

\[
\begin{array}{c|c|c}
\varepsilon = 0 & \varepsilon \neq 0 & \varepsilon = 0 \\
I & II & III \\
p_{po} \approx N_A & n_{po} \approx \frac{n_i^2}{N_A} & n_{no} \approx N_D \\
n_p & p_n & n_{no} \approx N_D \\
\end{array}
\]
Transit Time

- As an example, let’s consider the minority carries on the n-side.
- We will define the transit time as the time it takes for charges to go from accumulation to depletion.
- Depending on the physical nature of the pn junction and the operating conditions, $TT$ may correspond to the time it takes a carrier to transit the space charge region, or some sort of carrier recombination time.

\[ p_n \approx \frac{n_i^2}{N_D} \]

Diffusion Capacitance

- We can describe our diffusion charge:
  \[ q_{\text{diff}} = i_{\text{cond}} \times TT \]  \hspace{1cm} (9)
- We can then define the diffusion capacitance as:
  \[ C_{\text{diff}} = \frac{dq_{\text{diff}}}{dV_D} = TT \frac{di_{\text{cond}}}{dV_D} \]  \hspace{1cm} (10)
- We spent the last 2 lectures deriving an equation for current ($i$) as a function of voltage ($V_D$):
  \[ i = I_0 \left[ \exp \left( \frac{eV_D}{nk_B T} \right) - 1 \right] \]  \hspace{1cm} (11)
  \[ C_{\text{diff}} = \frac{TT eI_0}{nk_B T} \exp \left( \frac{eV_D}{nk_B T} \right) \]  \hspace{1cm} (12)
Diode Capacitance

- So we can now describe the total diode capacitance:
  \[ C_{\text{diode}} = C_{\text{depl}} + C_{\text{diff}} \]  

  \[ C_{\text{diode}} = C_{\text{depl}}(0) \left[ 1 - \frac{V_D}{\phi_{bi}} \right]^{-1/2} + \frac{TTeI_0}{nk_BT} \exp \left( \frac{eV_D}{nk_BT} \right) \]  

Due to majority carriers

- Before we put together the large signal equivalent circuit model for the pn junction, we should also account for series resistance associated with the ohmic contacts and the bulk regions of the pn junction.

Series Resistance

- We account for this by lumping into one lumped circuit series resistance parameter: \( R_s \).

- So this is our large-signal equivalent circuit model.
Small-Signal Equivalent Circuit

The idea is that we operate the device at a given DC voltage along the large-signal IV characteristic while simultaneously superimposing a small AC signal. Effectively, this linearizes device operation. Thus, small signal operation allows a highly non-linear device to be operated and analyzed as a linear device. This is useful both practically and analytically.
Small-Signal Equivalent Circuits

- To accomplish this nonlinear to linear transformation, begin with the nonlinear large-signal model and the corresponding device physics equations.
- Recall, for the previous large signal model:
  - \( R_s \) is already a linear device.
  - \( C_{\text{depl}} \) and \( C_{\text{diff}} \) are linearized to normal capacitors by recognizing that at a given DC operating point, these capacitances are constants.
  - \( i_{\text{cond}} \) is the nonlinear, voltage dependent current source.

Linearizing Current

- \( i_{\text{cond}} \) can be linearized by the following approach:
  - Taking the appropriate derivative with respect to the diode voltage.
  - Evaluating at the DC operating point.
  - Identifying the equivalent circuit element that arises.
- We know \( i_{\text{cond}} \) is described by:
  \[
  i_{\text{cond}} = I_0 \left[ \exp \left( \frac{eV_D}{nk_BT} \right) - 1 \right]
  \]  
  \( \text{(2)} \)
- How do we linearize this variable?
Small-Signal Equivalent Circuits

- We can instead describe this element as voltage dependent resistor:

\[ r_{\text{depl}} = \frac{n k_B T}{e i_{\text{cond}}(V_D)} \]  

- So this is our large-signal equivalent circuit model.

Fletcher’s Boundary Conditions
High Injection

• Recall, in our derivation of the Shockley Equation, we assumed that the injected minority carrier densities are low compared to majority carrier densities.

• Last time we talked about generalizations to this assumption in terms of series resistance and Fermi-Level splitting.

• However we still assumed the following:
  • That the quasi-Fermi levels are flat across the junction.
  • The majority carrier concentration changes negligibly at either side of the junction.

We derived the following equations describing charge density at the edge of the depletion region:

\[ p_n(x = W_{Dn}) = p_{n0} \exp \left( \frac{eV_D}{k_BT} \right) \]  (15)

\[ n_p(x = -W_{Dp}) = n_{p0} \exp \left( \frac{eV_D}{k_BT} \right) \]  (16)

• We call these the Shockley boundary conditions.
High Injection

- I.e. when we plot the Fermi Levels or carrier concentrations:

![Energy-band diagram](image)

<table>
<thead>
<tr>
<th>Forward Bias</th>
<th>Reverse Bias</th>
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<tbody>
<tr>
<td><img src="image" alt="Energy-band diagram" /></td>
<td><img src="image" alt="Energy-band diagram" /></td>
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Fletcher’s Boundary Conditions

- In real devices, there are quite a few situations in which high level injection prevails and the Shockley boundary conditions are not appropriate.
- In these situations we have to use more general (and slightly less user-friendly) boundary conditions.
- Normally we would use either:
  1). Fletcher boundary conditions.
  2). Misawa boundary conditions.
  3). Dirichlet boundary conditions.
  4). Neumann boundary conditions.

- Let’s start with the Fletcher boundary conditions.
Fletcher’s Boundary Conditions

• In Fletcher’s Boundary Conditions we still assume that the quasi-Fermi level is flat across the junction.
• Here let’s say under applied bias: $\phi_{bi} \rightarrow \phi_{bi} - V_j$

Fletcher’s Boundary Conditions

• We define $V_j$ to explicitly account for the fact that not all of the applied voltage is dropped across the junction (i.e. some of it may be dropped across the quasi-neutral regions and even the ohmic contacts if they are not ideal).
• From our diagram, we can express the minority carrier in terms of the majority carrier density at the boundaries:

$$p_n(W_{Dn}) = p_p(-W_{Dp}) \exp\left(-\frac{e\phi_{bi}}{k_BT}\right) \exp\left(\frac{eV_j}{k_BT}\right) \quad (17)$$

• The reduction is due to the fact that $E_F$ is $e(\phi_{bi} - V_j)$ further from $E_V$ on the n-side (by definition).
Fletcher’s Boundary Conditions

- Can deduce an equivalent expression for electrons:

\[ p_n(W_{Dn}) = p_p(-W_{Dp}) \exp\left(-\frac{e\phi_{bi}}{k_B T}\right) \exp\left(\frac{eV_j}{k_B T}\right) \]  \hspace{1cm} (17)

\[ n_p(-W_{Dp}) = n_n(W_{Dn}) \exp\left(-\frac{e\phi_{bi}}{k_B T}\right) \exp\left(\frac{eV_j}{k_B T}\right) \]  \hspace{1cm} (18)

- Note that \( p_p(-W_{Dp}) \) and \( n_n(W_{Dn}) \) are the non-equilibrium majority carrier concentrations at the edge of the depletion region.

- These values are not equal to the equilibrium majority carrier concentrations (\( n_{n0} \) and \( p_{p0} \)), since we are assuming that high-level injection occurs.

Fletcher’s Boundary Conditions

- Graphically you can think about the last point like this:

\[
\begin{align*}
\text{\( \varepsilon = 0 \)} & & \text{\( \varepsilon \neq 0 \)} & & \text{\( \varepsilon = 0 \)} \\
\text{I} & & \text{II} & & \text{III} \\

p \approx N_A & & p_p & & n \approx N_D \\
\frac{n_p}{n_{i0}} \approx N_A & & \frac{n_p}{n_{i0}} & & \frac{n_p}{n_{i0}} \approx \frac{n_{i0}^2}{N_D} \\
\frac{n_p}{n_{i0}} \approx \frac{n_{i0}^2}{N_A} & & \frac{n_p}{n_{i0}} & & \frac{n_p}{n_{i0}} \approx \frac{n_{i0}^2}{N_D}
\end{align*}
\]
Fletcher’s Boundary Conditions

- Using the following:
  \[ \phi_{Bi} = \frac{k_B T}{e} \ln \left( \frac{p_{p0}}{p_{n0}} \right) = \frac{k_B T}{e} \ln \left( \frac{n_{p0}}{n_{n0}} \right) \]  
  (19)

- We can re-write (17) and (18) as:
  \[ p_n(W_{Dn}) = p_p(-W_{Dp}) \frac{p_{p0}}{p_{n0}} \exp \left( \frac{eV_J}{k_B T} \right) \]  
  (20)

  \[ n_p(-W_{Dp}) = n_n(W_{Dn}) \frac{n_{p0}}{n_{n0}} \exp \left( \frac{eV_J}{k_B T} \right) \]  
  (21)

Fletcher’s Boundary Conditions

- Charge neutrality requires that the increase of the majority and minority carrier concentrations at the edge of the depletion region are equal.

- From this we can say:
  \[ p_p(-W_{Dp}) - p_{p0} = n_p(-W_{Dp}) - n_{p0} \]  
  (22)

  \[ n_n(W_{Dn}) - n_{n0} = p_n(W_{Dn}) - p_{n0} \]  
  (23)

- Equations (20-23) are sufficient to solve for the four unknown carrier concentrations at the edge of the depletion region: \( p_n(W_{Dn}), n_p(-W_{Dp}), p_p(-W_{Dp}), n_n(W_{Dn}) \).
Fletcher’s Boundary Conditions

- We will not solve these simultaneous equations, just quote the results:

\[
p_n(W_{Dn}) = \frac{(p_0 - n_0)n_0p_0e^{\frac{eV_J}{k_BT}} + (n_0 - p_0)n_0p_0e^{\frac{2eV_J}{k_BT}}}{n_0p_0 - n_0p_0e^{\frac{k_BT}{eV_J}}} \tag{24}
\]

\[
n_p(-W_{Dp}) = \frac{(n_0 - p_0)n_0p_0e^{\frac{eV_J}{k_BT}} + (p_0 - n_0)n_0p_0e^{\frac{2eV_J}{k_BT}}}{n_0p_0 - n_0p_0e^{\frac{k_BT}{eV_J}}} \tag{25}
\]

\[
p_p(-W_{Dp}) = \frac{(p_0 - n_0)n_0p_0 + (n_0 - p_0)n_0p_0e^{\frac{eV_J}{k_BT}}}{n_0p_0 - n_0p_0e^{\frac{k_BT}{eV_J}}} \tag{26}
\]

\[
n_n(W_{Dn}) = \frac{(n_0 - p_0)n_0p_0 + (p_0 - n_0)n_0p_0e^{\frac{eV_J}{k_BT}}}{n_0p_0 - n_0p_0e^{\frac{k_BT}{eV_J}}} \tag{27}
\]

- These are Fletcher’s Boundary Conditions.
Other Boundary Conditions

Misawa’s Boundary Conditions

- An alternate way to handle high level injection is the Misawa’s Boundary Conditions.
- One version of which can be expressed as:

\[ n_p(-W_{DP})p_p(-W_{DP}) = n_n(W_{DN})p_n(W_{DN}) \]

\[ = n_i^2 \exp \left[ \frac{E_{FP}(-W_{DP}) + E_{FN}(W_{DN})}{k_BT} \right] \]  

(28)

- Which looks very similar to Schockley’s law of the junction:

\[ n(x)p(x) = n_i^2 \exp \left[ \frac{E_{FP} + E_{FN}}{k_BT} \right] = n_i^2 \exp \left( \frac{eV_j}{k_BT} \right) \]

(29)
**Misawa’s Boundary Conditions**

- In the high-level injection situation, for which the Misawa BC's apply, the splitting of the quasi-Fermi levels across the junction is not necessarily equal to the applied voltage, since some of the applied voltage is also dropped across the quasi-neutral regions, and perhaps the Ohmic contacts.
  - I.e. the distinction between Misawa’s Boundary Conditions and Shockley’s Law of the Junction is **series resistance**.
- Although it is not obvious, it turns out that the Fletcher and Misawa B.C.'s are equivalent.

**Dirichlet Boundary Conditions**

- One form of the Dirichlet B.C.'s for a metal-semiconductor interface is given by the following:

\[
J_n = e S_n \left[ n - \frac{1}{2} \left( N + \sqrt{N^2 + 4n_i^2} \right) \right] \tag{30}
\]

\[
J_p = e S_p \left[ p - \frac{1}{2} \left( -N + \sqrt{N^2 + 4n_i^2} \right) \right] \tag{31}
\]

- \( S_n \) is the surface recombination velocity for electrons.
- \( S_p \) is the surface recombination velocity for holes.
- \( eN \) is the excess immobile charge at the interface.

\[
N = n - p \tag{32}
\]
Dirichlet Boundary Conditions

- Surface recombination velocity is a parameter which characterizes the likelihood of having carrier recombination at a surface.
- Note that if \( S_n \) and \( S_p \) are infinite, we can rearrange these equations to obtain:

\[
n = \frac{1}{2} \left( N + \sqrt{N^2 + 4n_i^2} \right) \quad (33)
\]

\[
p = \frac{1}{2} \left( -N + \sqrt{N^2 + 4n_i^2} \right) \quad (34)
\]

- Equations (33) and (34) constitute the Dirichlet Boundary Conditions for Ohmic contacts.
- Note that an ideal Ohmic contact has an infinite surface recombination velocity.
- This ensures that any excess carriers which reach the contact disappear.
- Recall that an Ohmic contact provides any required amount of carriers with a negligible amount of voltage drop across it.
Neumann Boundary Conditions

- Now consider the interface between a semiconductor and an insulator.
- The electrostatic Boundary Conditions (Gauss's Law) require that for fields normal to the interface:
  \[ \varepsilon_r \varepsilon_0 \varepsilon_s - \varepsilon_i \varepsilon_0 \varepsilon_i = Q_{\text{interface}} \] (35)
  - \( \varepsilon_r \) relative permittivity of the semiconductor.
  - \( \varepsilon_i \) relative permittivity of the interface.
  - \( \varepsilon_s \) electric field in the semiconductor.
  - \( \varepsilon_i \) electric field in the insulator.
  - \( Q_{\text{interface}} \) is charge / unit area at interface.

- If we assume the insulator is infinitely thick we say:
  \[ \varepsilon_r \varepsilon_0 \varepsilon_s = Q_{\text{interface}} \] (36)
- Furthermore, it is assumed that the interface charge is free:
  \[ \varepsilon_r \varepsilon_0 \varepsilon_s = 0 \]
- I.e. we have to say:
  \[ \varepsilon_s = 0 \] (37)
- Equation (37) is the Neumann Boundary Condition.
Surface Recombination

- Since we brought up surface recombination velocity in the context of Dirichlet Boundary Conditions we will delve a little further.
- If there is an enhanced concentration of recombination centers at a surface, you would expect an enhanced recombination rate there, such that there would be a smaller excess carrier concentration at the surface.
- In steady state, the e\(^-\) and h\(^+\) currents flowing to the surface must be equal and opposite in order to maintain a steady-state rate of surface recombination.
Surface Recombination

- We are saying that of the total trap density, the majority are located at the surface.

\[ N_T \]

\[ x_t \]

\[ x \]

\[ p \]

\[ p_n(0) \]

\[ p_n \]

\[ d \]

\[ x \]

\[ J_p \]

\[ e \]

\[ D_p \]

\[ \Delta p \]

\[ x = 0 \]

\[ J_p(x = 0) = -eD_p \frac{d\Delta p(0)}{dx} = eU_s = eS_p\Delta p(0) \] (38)

- \( J_p \) hole diffusion current density.
- \( D_p \) hole diffusion coefficient.
- \( \Delta p \) perturbation from equilibrium hole density:
  \[ \Delta p(x) = p_n(x) - p_n(0) \] (39)
- \( U_s \) net rate of surface recombination.
- \( S_p \) hole recombination velocity.
Surface Recombination

- $S_p$, the hole recombination velocity, is determined using SRH recombination theory (assuming a single, discrete trap) applied at the surface.

$$U_s = \frac{\sigma_n \sigma_p v_{th} N_{ts} (pn - n_t^2)}{\sigma_n (n + n_{ts}) + \sigma_p (p + p_{ts})} \quad (40)$$

- $U_s$ net rate of surface recombination.
- $\sigma_n$ is the electron capture cross section.
- $\sigma_p$ is the hole capture cross section.
- $N_{ts}$ is the density of surface traps per unit area at an energy $E_t$ below the conduction band.
- $v_{th}$ is the carrier thermal velocity.

$n_{ts}$ is the density of $e^-$ in the conduction band at the surface if $E_F$ is positioned at the trap energy.

$p_{ts}$ is the density of $h^+$ in the valence band at the surface if $E_F$ is positioned at the trap energy.

$$n_{ts} = n_i e^{(E_t - E_i)/k_BT} \quad (41)$$

$$p_{ts} = n_i e^{(E_i - E_t)/k_BT} \quad (42)$$
Summary

• We developed large and small signal equivalent circuits:

![Large and small signal equivalent circuits diagram]

- Boundary conditions under high injection:

![Boundary conditions diagram]

- Surface recombination:

![Surface recombination diagram]

Next Time...

• Time-Dependence of pn-Junctions.

![Time-Dependence of pn-Junctions diagram]

- Reading: Section 2.5 of Sze & Ng (p114-117).