CS 536: Introduction to Graphical Models  
Winter 2020  
Assignment #3

Out: Wednesday, Jan 22, 2020  
Due: In class, Wednesday, Jan 29, 2020  
Total marks: 35

1. (Exercise 4.1 in the book) Complete the analysis of example 4.4, showing that the distribution $P$ defined in the example does not factorize over $\mathcal{H}$. (Hint: Use a proof by contradiction). If you read Theorem 4.1 to Example 4.4 on pages 115-116, it will really help with this question. [10 points]

2. (Exercise 4.10 in the book) We define the following properties for a set of independencies:
   - Strong Union: $(X \perp Y|Z) \implies (X \perp Y|Z, W)$
     In other words, additional evidence $W$ cannot induce dependence
   - Transitivity: For all disjoint sets $X, Y, Z$ and all variables $A$ (where $A$ is not a part of $X, Y,$ or $Z$):
     \[ \neg(X \perp A|Z) \& \neg(A \perp Y|Z) \implies \neg(X \perp Y|Z) \]
     Intuitively, this statement asserts that if $X$ and $Y$ are both correlated with some $A$ (given $Z$), then they are also correlated with each other (given $Z$). We can also write the contrapositive of this statement, which is less obvious but easier to read.
     For all $X, Y, Z, A$:
     \[ (X \perp Y|Z) \rightarrow (X \perp A|Z) \lor (A \perp Y|Z). \]
     Prove that if $\mathcal{I} = I(H)$ for some Markov network $H$, then $\mathcal{I}$ satisfies strong union and transitivity. (If you aren’t familiar with some of the symbols, $\neg$ means logical NOT, $\&$ means logical AND and $\lor$ means logical OR) [10 points]

3. (Exercise 4.14a in the book). The Markov blanket of a node $X$ in a Bayesian network $G$, denoted $\text{MB}_G(X)$, is defined to be the nodes consisting of $X$’s parents, $X$’s children, and other parents of $X$’s children. Show the following:
   For any variable $X$, let $W = X - \{X\} - \text{MB}_G(X)$ where $X$ is the set of all random variables in the Bayesian network $G$. Then $d$-sep$_G(X; W | \text{MB}_G(X))$ [10 points]

4. (Exercise 4.18 in the book) Let $G$ be a Bayesian network structure and $\mathcal{H}$ a Markov network structure over $\mathcal{X}$ such that the skeleton of $G$ is precisely $\mathcal{H}$. Prove that if $G$ has no immoralities, then $I(G) = I(\mathcal{H})$. [5 points]