

Bayesian Networks 3

D-separation

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D-separation

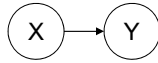
- Given a graph \mathcal{G} , we would like to “read off” independencies
- The converse is easier to think about: when does an independence statement not hold?
- Eg. when can X influence Y ?

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D-separation

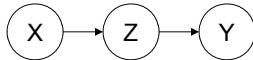
When can X influence Y ?

Direct connection:

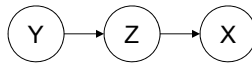


Indirect connection:

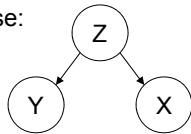
Indirect causal effect:



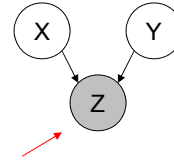
Indirect evidential effect:



Common cause:



Common effect
(v-structure):

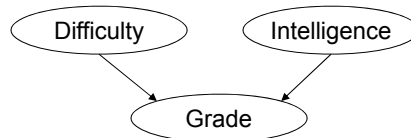


Note: Z is observed as evidence

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Explaining Away

Let's take a closer look at the common effect case
(also known as explaining away):



When Grade is not observed:

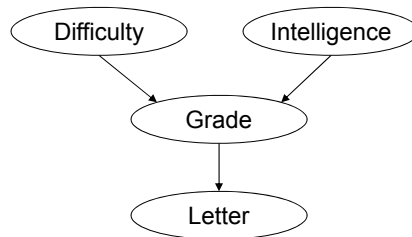
- You can't really say anything about the Intelligence of the student given the Difficulty of the course

When Grade is observed eg. a C:

- Difficulty and Intelligence are **not** independent
- Eg. if we observe Grade = C and Difficulty = Low, we tend to believe Intelligence = Low

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D-separation



What happens if we observe Letter = weak as evidence?

- Indicates that student had a low Grade
- Intelligence and Difficulty are now not independent (as in the previous slide)

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D-separation

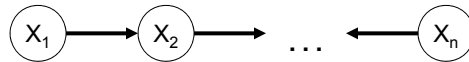
When influence can flow from X to Y via Z, we say that the trail $X \leftrightarrow Z \leftrightarrow Y$ is **active** (otherwise it is **blocked**):

- | | | |
|-------------------|--|--|
| Causal trail: | | Active if and only if Z is not observed |
| Evidential trail: | | Active if and only if Z is not observed |
| Common cause: | | Active if and only if Z is not observed |
| Common effect: | | Active if and only if either Z or one of Z's descendants is observed |

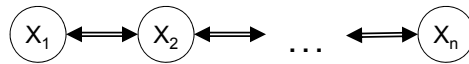
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D-separation

All the previous cases deal with 3 node trails.
Suppose we have a longer trail:



First, ignore the arrows. We will designate that we don't care about the arrow direction by using



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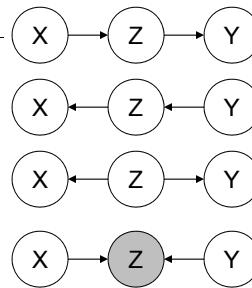
D-separation

For influence to flow from X_1 to X_n , it needs every two-edge trail along the trail to allow influence to flow ie.

Take



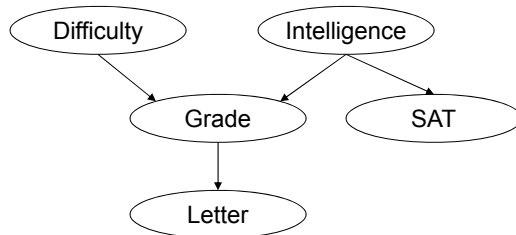
Put the original arrows back in, and it must match the patterns on the right



Or one of the descendants of Z is observed

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D-separation



(Examples) Consider the trail $D \rightarrow G \leftarrow I \rightarrow S$

- If $Z = \{ \}$, the trail is not active ($D \rightarrow G \leftarrow I$ not active)
- If $Z = \{L\}$ the trail is active
- If $Z = \{L, I\}$ the trail is not active (I blocks the trail $G \leftarrow I \rightarrow S$)⁹

D-separation

- **D-separation**: Let X , Y , Z be three sets of nodes in \mathcal{G} . We say that X and Y are d-separated given Z , denoted $d\text{-sep}_{\mathcal{G}}(X; Y | Z)$, if there is no active trail between any node $X \in X$ and $Y \in Y$ given Z .
- Use $I(\mathcal{G})$ to denote the set of independencies that correspond to d-separation:
$$I(\mathcal{G}) = \{(X \perp Y | Z) : d\text{-sep}_{\mathcal{G}}(X; Y | Z)\}$$
- This set is also called the set of **global Markov independencies**

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D-separation Exercises

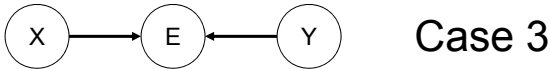
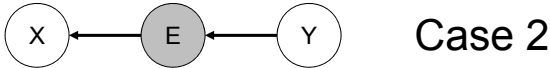
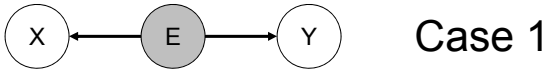
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D-separation Recipe

- To determine if $(X \perp Y \mid E)$, ignore the directions of the arrows, find all paths between X and Y
- Now pay attention to the arrows. Determine if the paths are blocked according to the 3 cases
- If all the paths are blocked, X and Y are d-separated given E
- Which means they are conditionally independent given E

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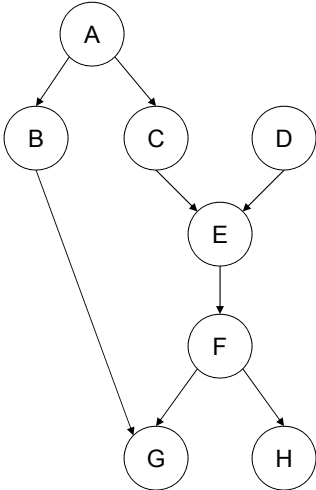
Blocked Paths



Note: E is not observed

D-separation Examples

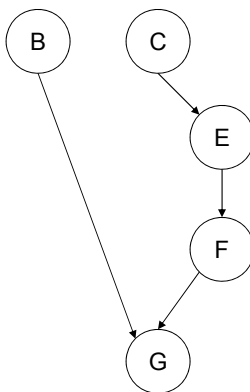
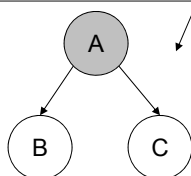
$(B \perp C | A)?$



D-separation Examples

$(B \perp C \mid A)$? Yes. Notice the two (undirected) paths between B and C

This path from B to C is blocked by A (Case 1)

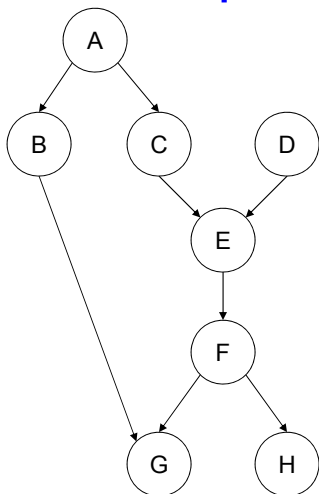


This path from B to C is blocked by G, which is not in the evidence set (Case 3)

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D-separation Examples

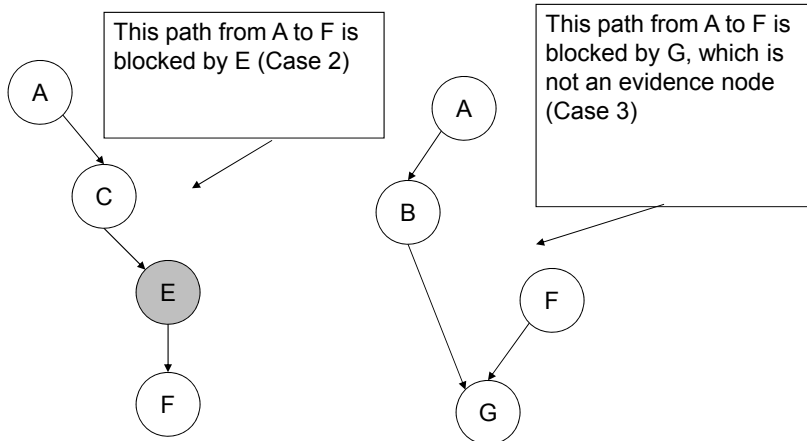
$(A \perp F \mid E)$?



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D-separation Examples

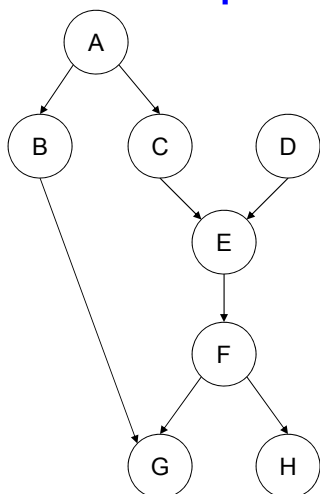
$(A \perp F \mid E)$? Yes



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D-separation Examples

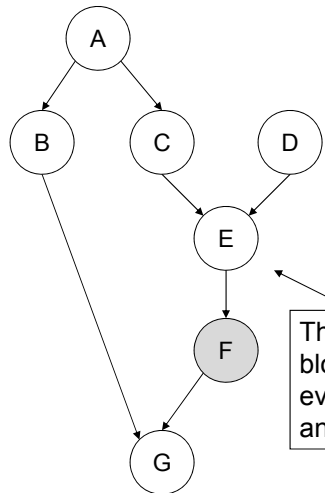
$(C \perp D \mid F)$?



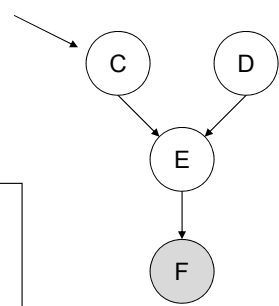
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D-separation Examples

$(C \perp\!\!\!\perp D \mid F)$? No



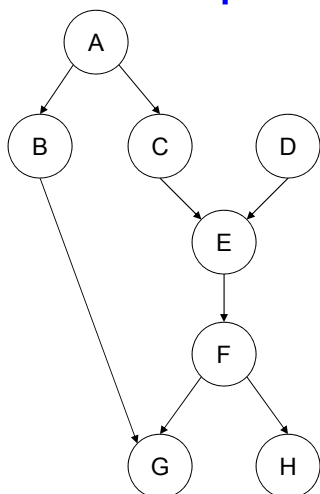
But this path from C to D is **not** blocked. This is because F (which is a descendant of E) is in the evidence set (Case 3)



This path from C to D is blocked by G (not in evidence set) (Case 3) and by F (Case 2)

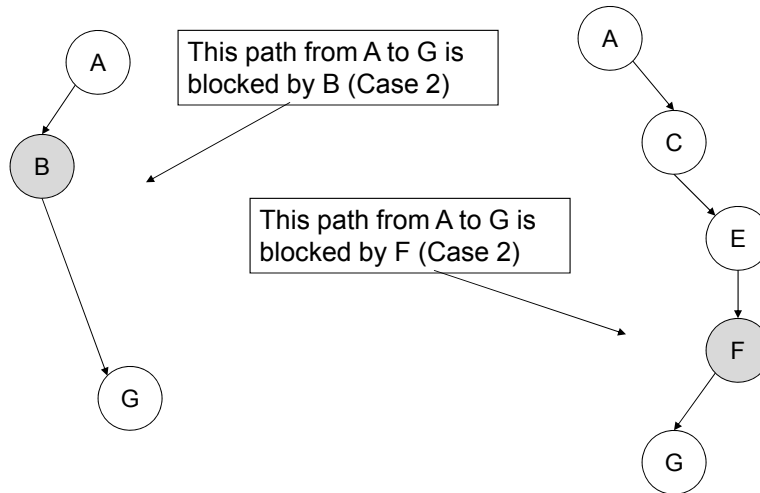
D-separation Examples

$(A \perp\!\!\!\perp G \mid \{B, F\})$?



D-separation Examples

$(A \perp G \mid \{B, F\})$? Yes



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