Bayesian Networks 3
D-separation

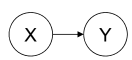
D-separation

- Given a graph $\mathcal{G}$, we would like to "read off" independencies
- The converse is easier to think about: when does an independence statement not hold?
- Eg. when can ***X*** influence ***Y***?

D-separation

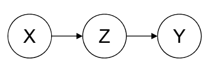
When can ***X*** influence ***Y***?

Direct connection:

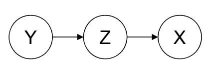


Indirect connection:

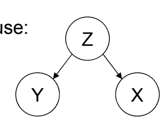
Indirect causal effect:



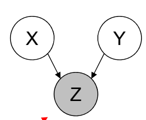
Indirect evidential effect:



Common cause:



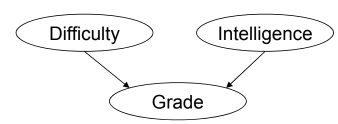
Common effect (v-structure):



Note: Z is observed as evidence

Explaining Away

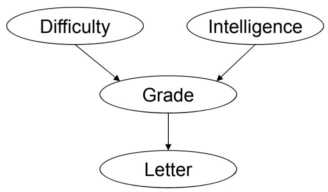
Let's take a closer look at the common effect case (also known as explaining away):



When Grade is <u>not observed</u>:
- You can't really say anything about the Intelligence of the student given the Difficulty of the course

When Grade is <u>observed</u> eg. a C:
- Difficulty and Intelligence are **not** independent
- Eg. if we observe Grade = C and Difficulty = Low, we tend to believe Intelligence = Low

D-separation



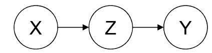
What happens if we observe Letter = weak as evidence?

- Indicates that student had a low Grade
- Intelligence and Difficulty are now not independent (as in the previous slide)

D-separation

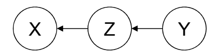
When influence can flow from X to Y via Z, we say that the trail X ⇔Z ⇔ Y is active (otherwise it is blocked):

Causal trail:



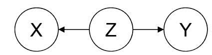
Active if and only if Z is not observed

Evidential trail:



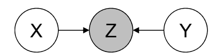
Active if and only if Z is not observed

Common cause:



Active if and only if Z is not observed

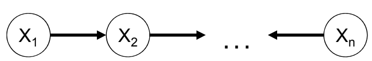
Common effect:



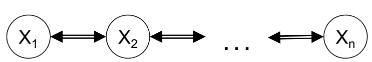
Active if and only if either Z or one of Z's descendants is observed

D-separation

All the previous cases deal with 3 node trails. Suppose we have a longer trail:



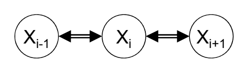
First, ignore the arrows. We will designate that we don't care about the arrow direction by using



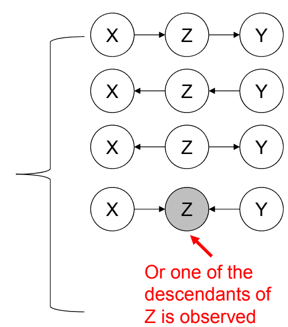
D-separation

For influence to flow from X_1 to X_n, it needs every two-edge trail along the trail to allow influence to flow ie.

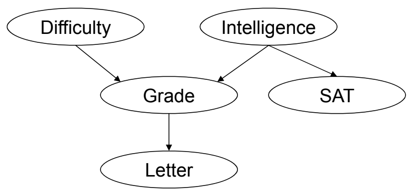
Take



Put the original arrows back in, and it must match the patterns on the right



D-separation



(Examples) Consider the trail D→G←I→S

- If Z={ }, the trail is not active (D→G←I not active)
- If Z={L} the trail is active
- If Z={L,I} the trail is not active (I blocks the trail G←I→S)

D-separation

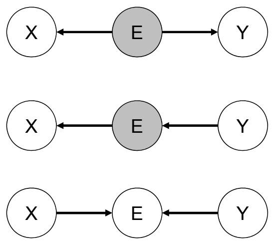
- D-separation: Let $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{Z}$ be three sets of nodes in $\mathcal{G}$. We say that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are d-separated given $\boldsymbol{Z}$, denoted d-sep$_{\mathcal{G}}$($\boldsymbol{X}$; $\boldsymbol{Y}$ | $\boldsymbol{Z}$), if there is no active trail between any node $X \in \boldsymbol{X}$ and $Y \in \boldsymbol{Y}$ given $\boldsymbol{Z}$.
- Use $\mathcal{I}(\mathcal{G})$ to denote the set of independencies that correspond to d-separation:
  $\mathcal{I}(\mathcal{G}) = \{(\boldsymbol{X} \perp \boldsymbol{Y} \mid \boldsymbol{Z}) : \text{d-sep}_{\mathcal{G}}(\boldsymbol{X}; \boldsymbol{Y} \mid \mathbf{Z})\}$
- This set is also called the set of global Markov independencies

D-separation Exercises

D-separation Recipe

- To determine if (X ⊥ Y | E), ignore the directions of the arrows, find all paths between X and Y
- Now pay attention to the arrows. Determine if the paths are blocked according to the 3 cases
- If all the paths are blocked, X and Y are d-separated given E
- Which means they are conditionally independent given E

Blocked Paths



Case 1

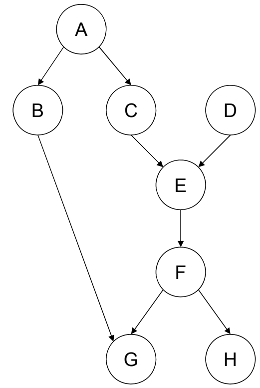
Case 2

Case 3

Note: E is not observed

13

D-separation Examples



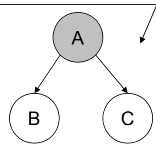
(B ⊥ C | A)?

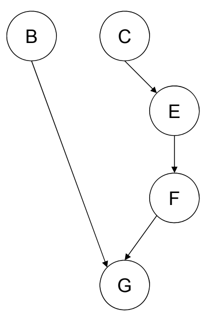
14

D-separation Examples

(B ⊥ C | A)? Yes. Notice the two (undirected) paths between B and C

This path from B to C is blocked by A (Case 1)

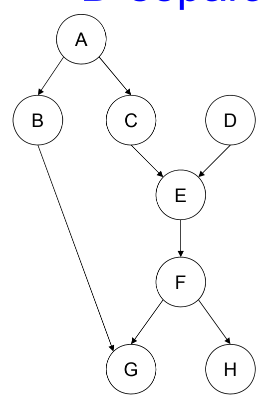




This path from B to C is blocked by G, which is not in the evidence set (Case 3)

15

D-separation Examples



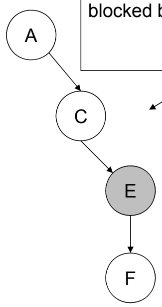
(A ⊥ F | E)?

16

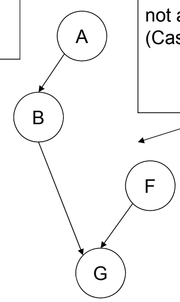
D-separation Examples

(A ⊥ F | E)? Yes

This path from A to F is blocked by E (Case 2)

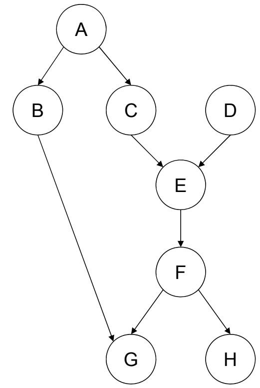


This path from A to F is blocked by G, which is not an evidence node (Case 3)



17

D-separation Examples

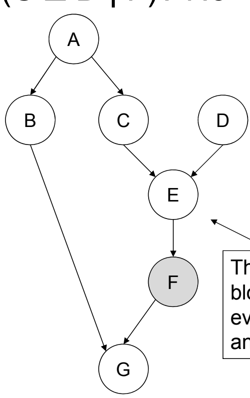


(C ⊥ D | F)?

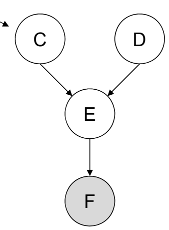
18

D-separation Examples

(C ⊥ D | F)? No



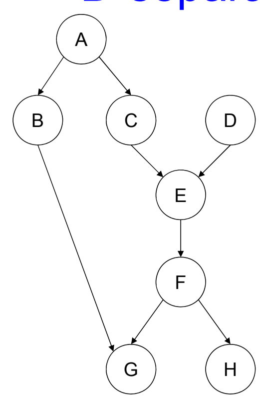
But this path from C to D is **not** blocked. This is because F (which is a descendant of E) is in the evidence set (Case 3)



This path from C to D is blocked by G (not in evidence set) (Case 3) and by F (Case 2)

19

D-separation Examples

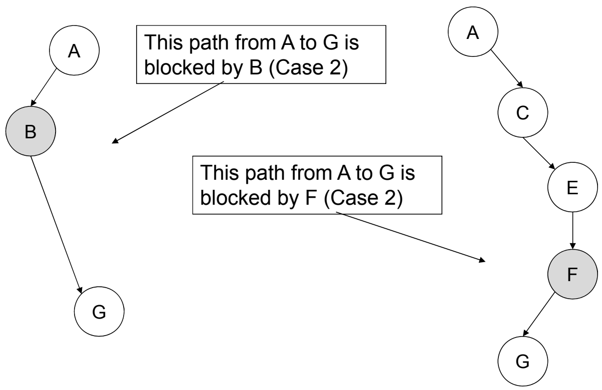


(A ⊥ G | {B, F})?

20

D-separation Examples

($A \perp G \mid \{B, F\}$)? Yes



21