ECE 464/564: Digital Signal Processing - Winter 2020
Homework 1

Due: Jan 21, 2020 (Tuesday)

1. The continuous-time signal

$$x_c(t) = \cos(40\pi t) + \sin(160\pi t)$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \cos\left(\frac{\pi n}{15}\right) + \sin\left(\frac{4\pi n}{15}\right).$$

(a) Determine a choice for T consistent with this information.

(b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

2. The continuous-time signal

$$x_c(t) = \frac{\cos(4000\pi t)}{4000\pi t}$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \frac{\cos(\pi n/3)}{(\pi n/3)}.$$

(a) Determine a choice for T consistent with this information.

(b) Is your choice for T in part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

3. Use the system shown in Fig. 1 below to implement a bandstop filter: $(\Omega_b T < \pi)$

$$H_c(j\Omega) = \begin{cases} 0 & \Omega_a \le |\Omega| \le \Omega_b \\ 1 & \text{otherwise.} \end{cases}$$

(C/D: An ideal continuous-to-discrete time converter, D/C: An ideal discrete-to-continuous time converter)

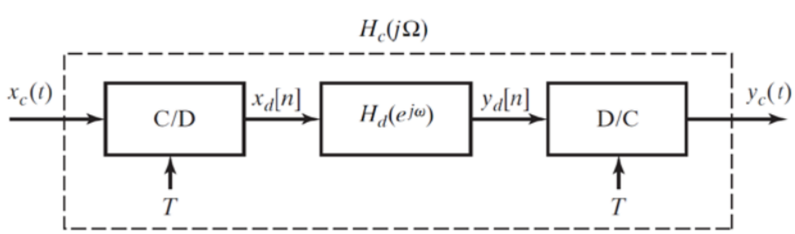

Figure 1: Continous-time filter using a discrete-time LPF.

(a) Find $H_d(e^{j\omega})$.

(b) Find $h[n]$ from $H_d(e^{j\omega})$.

4. Each of the following parts lists an input signal $x[n]$ and the Up-sampling and Down-sampling rates L and M, respectively, for the system in Fig. 2. Determine the corresponding output $\tilde{x}_d[n]$.

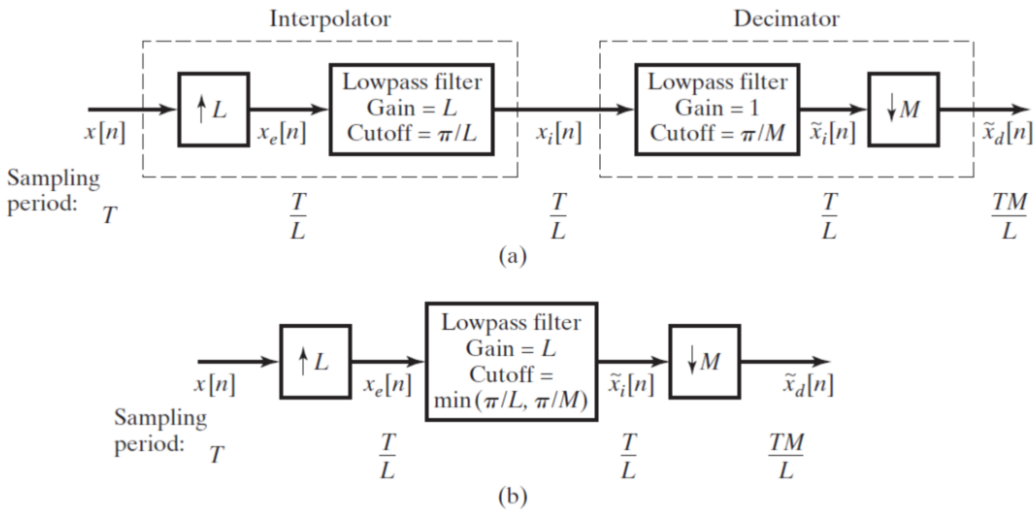

Figure 2: System for changing the sampling rate.

(a) $x[n] = \sin(4\pi n/5)/(\pi n), L = 6, M = 5.$

(b) $x[n] = \sin(2\pi n/3), L = 5, M = 6.$

5. For the system shown in Fig. 2, $X(e^{j\omega})$, the Fourier transform of the input signal $x[n]$, is shown in Fig. 3.

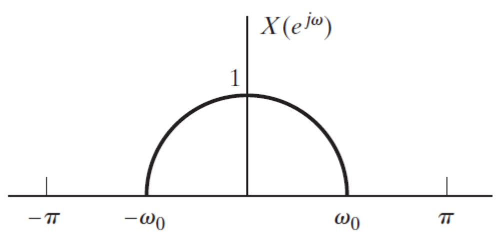

Figure 3: For problem 5.

For each of the following choices of L and M, specify the maximum possible value of ω_0 such that $X(e^{j\omega})$ can be recovered from $X_d(e^{j\omega})$, i.e., that $X(e^{jw})$ does not undergo any distortion effect that prevents it from being perfectly reconstructed from $X_d(e^{jw})$.

(a) $L = 8, M = 4.$

(b) $L = 4, M = 8.$

6. Consider a continous-time filter system (as in Fig. 1) that uses an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \leq \omega \leq \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi. \end{cases}$$

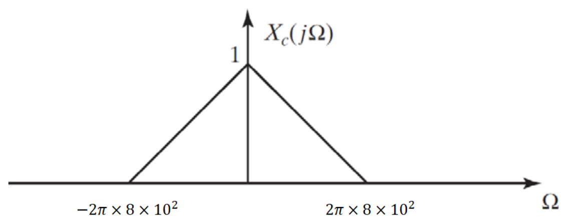

Figure 4: Continuous-Time Fourier Transform of $x_c(t)$.

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Fig. 4 and $\omega_c = 0.3\pi$, sketch and label $X(e^{j\omega}), Y(e^{j\omega})$ and $Y_c(j\Omega)$ for $T = 5 \times 10^{-4}$.

(b) For $T = 10^{-3}$ and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 8 \times 10^2$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which no aliasing occurs. For this maximum choice of ω_c, specify $H_c(j\Omega)$.