ECE 464/564 : HW 2 Solution

① a)

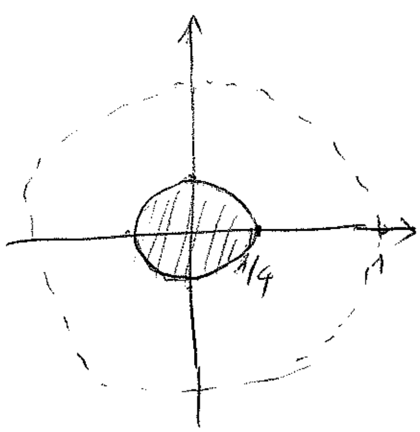
$$Y(z)\left(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right) = X(z)\left(1-\frac{3}{8}z^{-1}\right)$$

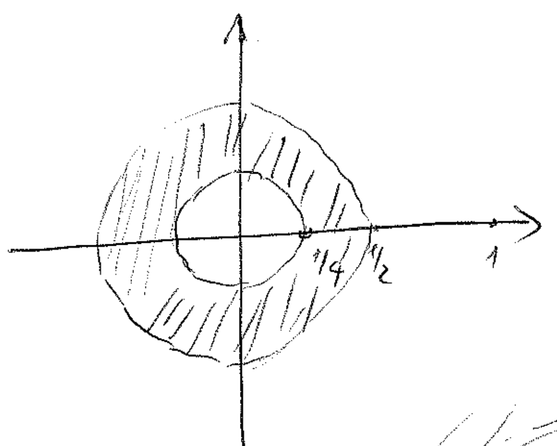
$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{3}{8}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

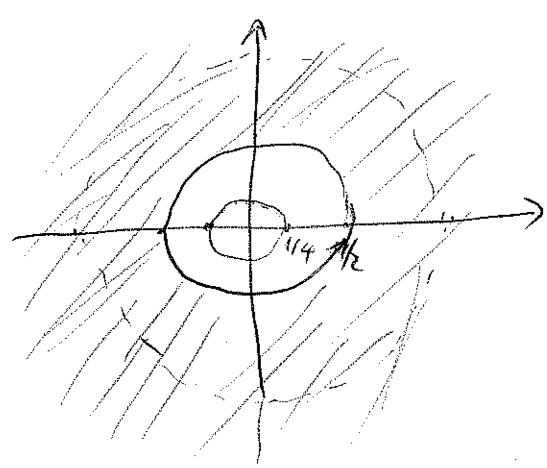
b)

$$H(z) = \frac{1-\frac{3}{8}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} = \frac{1/2}{1-\frac{1}{2}z^{-1}} + \frac{1/2}{1-\frac{1}{4}z^{-1}}$$

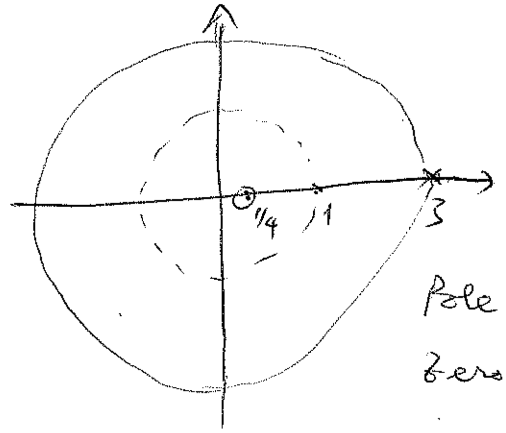
- For $|z| < \frac{1}{4}$: $h[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[-n-1] - \frac{1}{2}\left(\frac{1}{4}\right)^n u[-n-1]$
- For $\frac{1}{4} < |z| < \frac{1}{2}$: $h[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n u[n] - \frac{1}{2}\left(\frac{1}{2}\right)^n u[-n-1]$
- For $|z| > \frac{1}{2}$: $h[n] = \frac{1}{2}\left(\frac{1}{4}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^n u[n]$







(2) $X_1(z) = \dfrac{1 - \frac{1}{4}z^{-1}}{1 - 3z^{-1}}$



Pole : 3

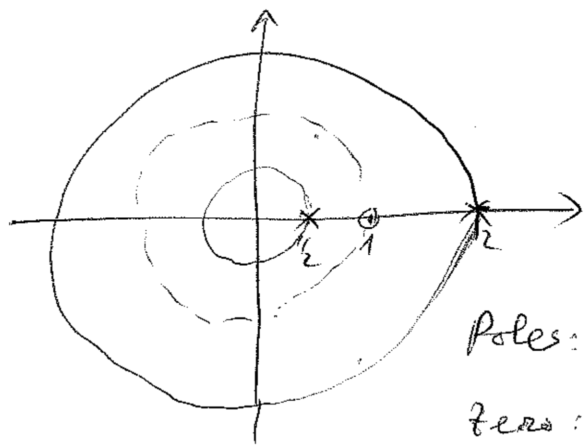
Zero : 1/4

+) $X_1(z)$ is stable for $|z| < 3$

+) $X_1(z)$ is causal for $|z| > 3$

+) ~~There~~ is no ROC where $X_1(z)$ is both stable and causal.

$X_2(z) = \dfrac{1 - z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$



Poles: $\frac{1}{2}$, 2

Zero : 1

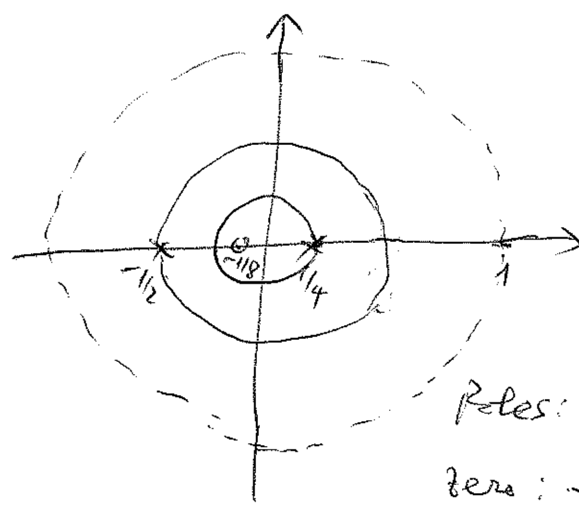
+) $X_2(z)$ is stable for $\frac{1}{2} < |z| < 2$

+) $X_2(z)$ is causal for $|z| > 2$

+) There is no ROC where $X_2(z)$ is both stable and causal

$X_3(z) = \dfrac{2(1 + 1/8\, z^{-1})}{(1 + 1/2\, z^{-1})(1 - 1/4\, z^{-1})}$

$X_3(z) = \dfrac{1}{1 + \frac{1}{2}z^{-1}} + \dfrac{1}{1 - \frac{1}{4}z^{-1}}$



Poles: $-\frac{1}{2}, \frac{1}{4}$

Zero : $-\frac{1}{8}$

+) $X_3(z)$ is stable for $|z| > \frac{1}{2}$

+) $X_3(z)$ is causal for $|z| > \frac{1}{2}$

+) $X_3(z)$ is both stable and causal for $|z| > \frac{1}{2}$

(3) Let us consider the general case in which $h[n]$ is symmetric around $\frac{n_0}{2}$. Then (similar for anti-symmetric)

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} h[\underbrace{n_0 - n}_{m}]\, e^{-j\omega n}$$

$$= \sum_{m=+\infty}^{-\infty} h[m]\, e^{-j\omega (n_0 - m)}$$

$$= \sum_{m=-\infty}^{+\infty} h[m]\, e^{-j\omega n_0}\, e^{j\omega m}$$

$$= e^{-j\omega n_0} \cdot H(e^{-j\omega}) = e^{-j\omega n_0} H^*(e^{j\omega})$$

$$\Rightarrow \measuredangle H(e^{j\omega}) = -\omega n_0 - \measuredangle H(e^{j\omega})$$

$$\Rightarrow \measuredangle H(e^{j\omega}) = -\frac{\omega n_0}{2}$$

$$\Rightarrow \text{grd}\left(H(e^{j\omega})\right) = -\frac{d \measuredangle H(e^{j\omega})}{d\omega} = \frac{n_0}{2}$$

<u>Thus</u>:

$\text{grd}\left(H_1(e^{j\omega})\right) = 0.5$

$\text{grd}\left(H_2(e^{j\omega})\right) = 2.5$

$\text{grd}\left(H_3(e^{j\omega})\right) = 1.5$

$\text{grd}\left(H_4(e^{j\omega})\right) = 5.5$

$\text{grd}\left(H_5(e^{j\omega})\right) = 2$

$\text{grd}\left(H_6(e^{j\omega})\right) = 0$

(4). From Fig. 2, we see that there are 3 narrow band signals

	ω	Gain	Group delay	Center
Signal 1	0.2π	10	150	$n \approx 80$
Signal 2	0.4π	1.4	0	$n \approx 30$
Signal 3	0.6π	0	-100	$n \approx 150$

Signal 3 disappears after the filter

Thus $y_3[n]$ in Fig. 3 can be the only possible solution