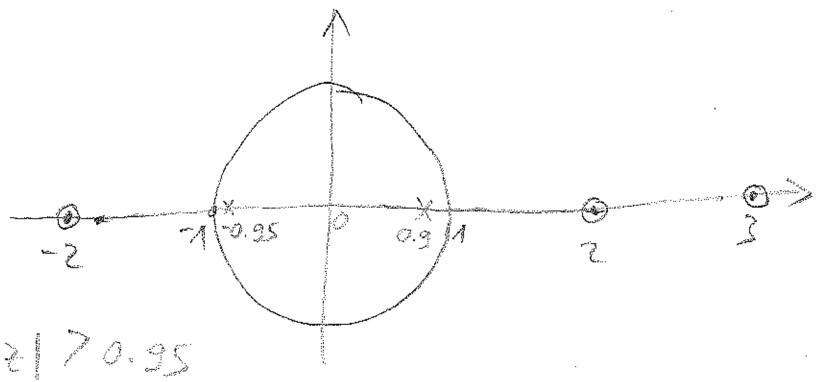
HW4 Solution

① $$H(z) = \frac{(1-2z^{-1})(1+2z^{-1})(1-3z^{-1})}{(1-0.9z^{-1})(1+0.95z^{-1})}$$

a/ Zeros: 2, −2, 3

Poles: 0.9, −0.95



The system is stable for $|z| > 0.95$

b/ $$H(z) = \underbrace{\frac{(1-2z^{-1})(1+2z^{-1})(1-3z^{-1})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}}_{H_{ap}(z)} \cdot \underbrace{\frac{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}{(1-0.9z^{-1})(1+0.95z^{-1})}}_{H_1(z)}$$

(2)

a/ False:

Parallel connection results in $H(z) = H_1(z) + H_2(z)$

A counterexample could be:

$H_1(z) = 1 + \frac{1}{2} z^{-1}$; $H_2(z) = c\left(1 - \frac{1}{4} z^{-1}\right)$

Then $H(z) = (c+1) - \left(\frac{c}{4} - \frac{1}{2}\right) z^{-1}$ has a zero at $\frac{c-2}{4c+4}$

Choosing $c = -3/2$ yields $\frac{c-2}{4c+4} = -\frac{7}{4}$ $\Rightarrow$ not a min. phase

b/ True:

Cascade connection results in $H(z) = H_1(z) \cdot H_2(z)$, which indicates all poles/zeros of $H(z)$ are either poles/zeros of $H_1(z)$ or $H_2(z)$. Thus, those will lie within the unit circle.

③ There can be multiple choices for $H_{min1}(z)$ and $H_{ap}(z)$.

a/ $$H(z) = \underbrace{\frac{z^{-1} - \frac{1}{3}}{1 - \frac{1}{3}z^{-1}}}_{H_{ap1}(z)} \cdot \underbrace{\frac{-3(1-\frac{1}{3}z^{-1})(1-0.2z^{-1})}{z^{-2}(1-0.5z^{-1})(1+0.5z^{-1})}}_{H_{min1}(z)}$$

b/ $$H(z) = \underbrace{\frac{z^{-1} - \frac{1}{3}}{z^{-1}(1 - \frac{1}{3}z^{-1})}}_{H_{ap2}(z)} \cdot \underbrace{\frac{-3(1-\frac{1}{3}z^{-1})(1-0.2z^{-1})}{z^{-1}(1-0.5z^{-1})(1+0.5z^{-1})}}_{H_{min2}(z)}$$

(4)

a/ $$H(z) = \frac{z^{-10}\left(\frac{\sqrt{2}}{2}e^{j\frac{3\pi}{4}} - z^{-1}\right)\left(\frac{\sqrt{2}}{2}e^{-j\frac{3\pi}{4}} - z^{-1}\right)}{\left(1 - \frac{\sqrt{2}}{2}e^{-j\frac{3\pi}{4}}z^{-1}\right)\left(1 - \frac{\sqrt{2}}{2}e^{j\frac{3\pi}{4}}z^{-1}\right)}$$

$$\Rightarrow |H(e^{j\omega})| = |e^{-j\omega \cdot 10}| \cdot |H_1(z)|$$

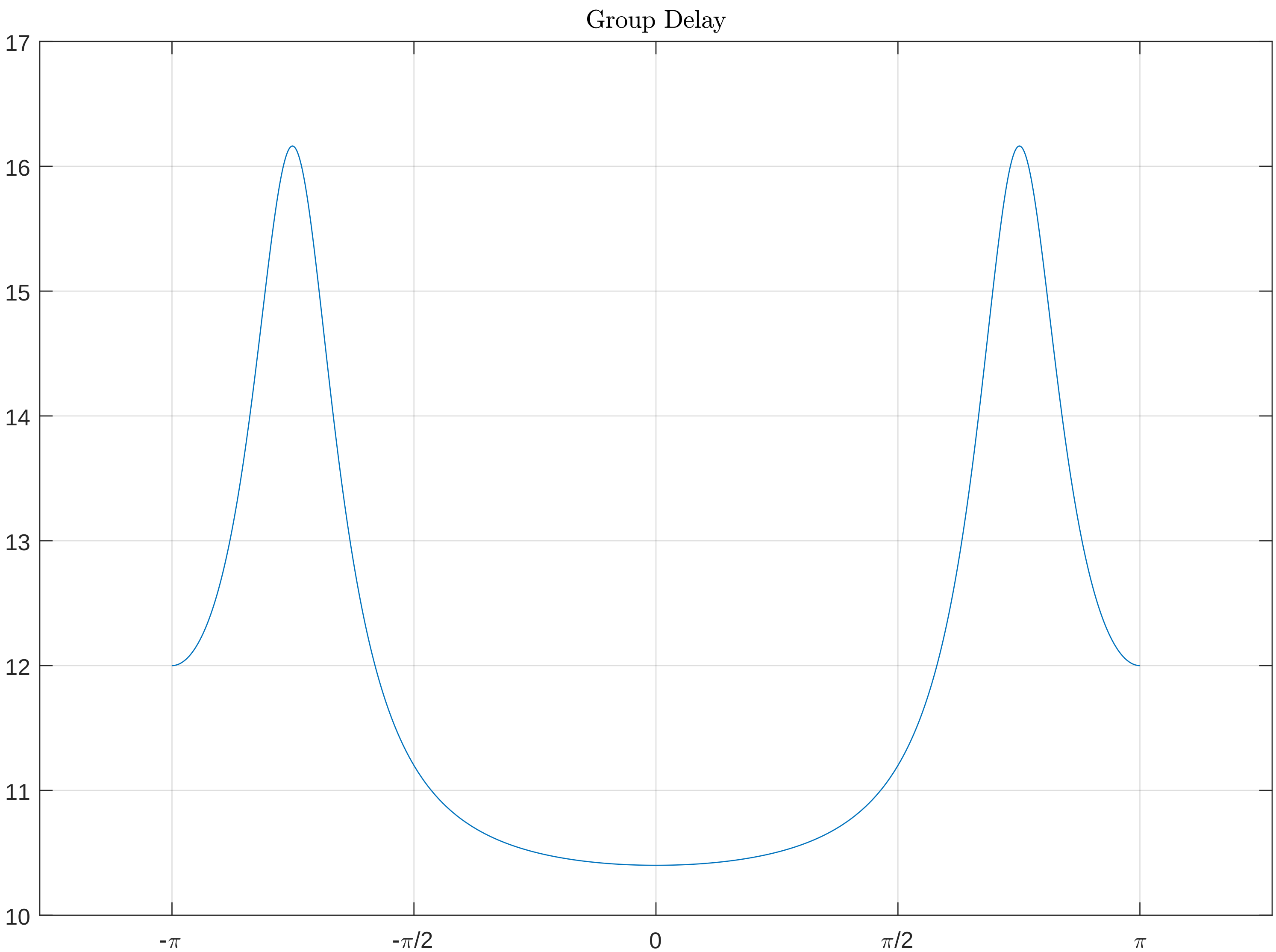
where $H_1(z) = \dfrac{\left(\frac{\sqrt{2}}{2}e^{j3\pi/4} - z^{-1}\right)\left(\frac{\sqrt{2}}{2}e^{-j3\pi/4} - z^{-1}\right)}{\left(1 - \frac{\sqrt{2}}{2}e^{-j\frac{3\pi}{4}}z^{-1}\right)\left(1 - \frac{\sqrt{2}}{2}e^{j3\pi/4}z^{-1}\right)}$ is an all-pass system $\left(r = \frac{\sqrt{2}}{2},\ \theta = \pm\frac{3\pi}{4}\right)$.

$\Rightarrow |H(e^{j\omega})| = 1 \Rightarrow H(z)$ is an all-pass system.

b/ $$\mathrm{grd}(H(z)) = \mathrm{grd}(e^{-j\omega \cdot 10}) + \mathrm{grd}(H_1(z))$$

$$= 10 + \frac{1}{3 - 2\sqrt{2}\cos\left(\omega - \frac{3\pi}{4}\right)} + \frac{1}{3 - 2\sqrt{2}\cos\left(\omega + \frac{3\pi}{4}\right)}$$

(see plot next page)


Group Delay
17
16
15
14
13
12
11
10
-π
-π/2
0
π/2
π