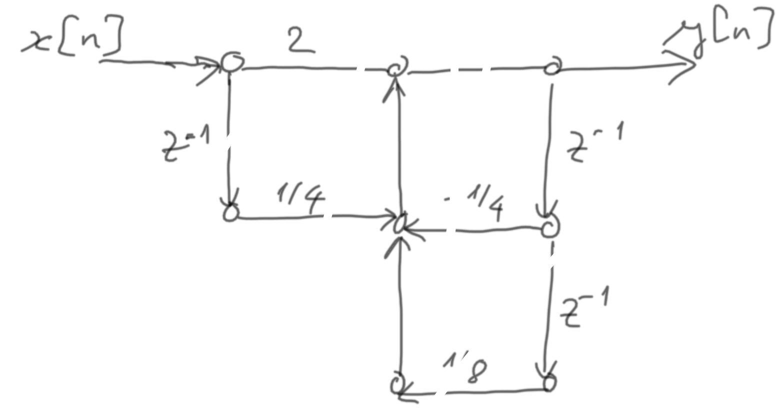
# HW 6 - Solution

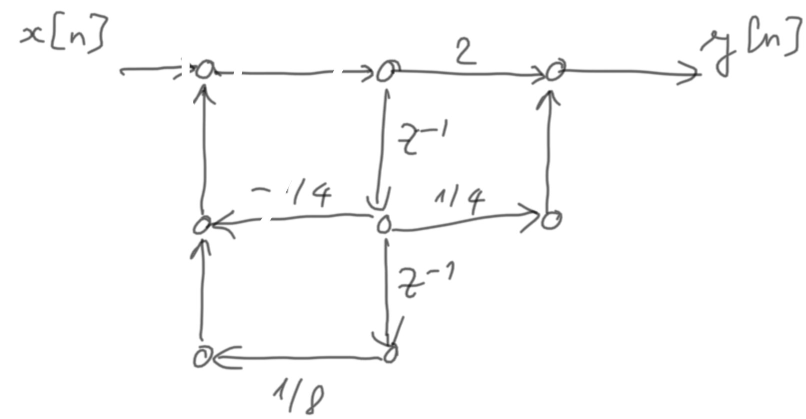
1

$$H(z) = \frac{2 + \frac{1}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{Y(z)}{X(z)}$$

Direct form I :




Direct form II :




2

a/
$$\begin{cases} w[n] = -\frac{1}{6}x[n] + \frac{1}{8}y[n] \\ v[n] = \frac{1}{6}x[n] + \frac{1}{4}y[n] + w[n-1] \\ y[n] = x[n] + \underbrace{v[n-1]} \end{cases}$$

b/ $y[n] = x[n] + \frac{1}{6}x[n-1] + \frac{1}{4}y[n-1] + \underbrace{w[n-2]}_{-\frac{1}{6}x[n-2] + \frac{1}{8}y[n-2]}$

$$y[n] = x[n] + \frac{1}{6}x[n-1] - \frac{1}{6}x[n-2] + \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2]$$

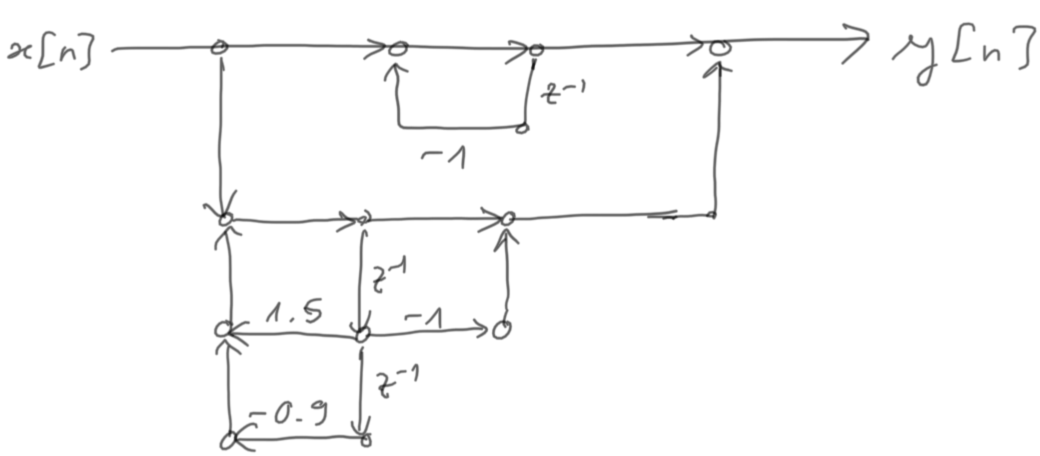
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$= \underbrace{\frac{(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})}}_{H_1(z)} \underbrace{\frac{(1-\frac{1}{3}z^{-1})}{(1+\frac{1}{4}z^{-1})}}_{H_2(z)}$$


c/ Yes. All poles/zeros are inside the unit circle

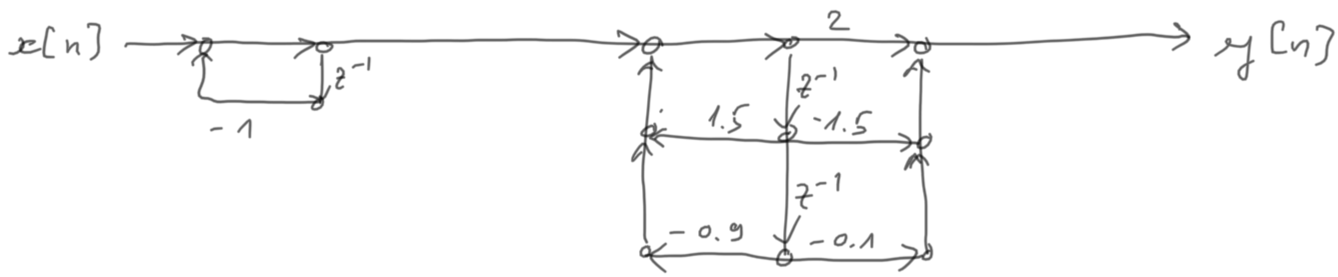
3 a/ No. The system has a pole $z = -1$.

b/

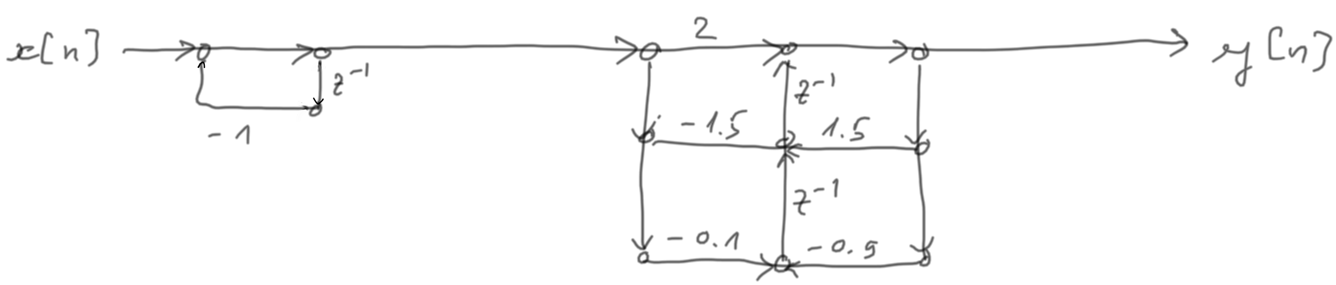


c/ $$H(z) = \frac{2 - 1.5z^{-1} - 0.1z^{-2}}{(1+z^{-1})(1 - 1.5z^{-1} + 0.9z^{-2})}$$

$$= \frac{1}{1+z^{-1}} \cdot \frac{2 - 1.5z^{-1} - 0.1z^{-2}}{1 - 1.5z^{-1} + 0.9z^{-2}}$$

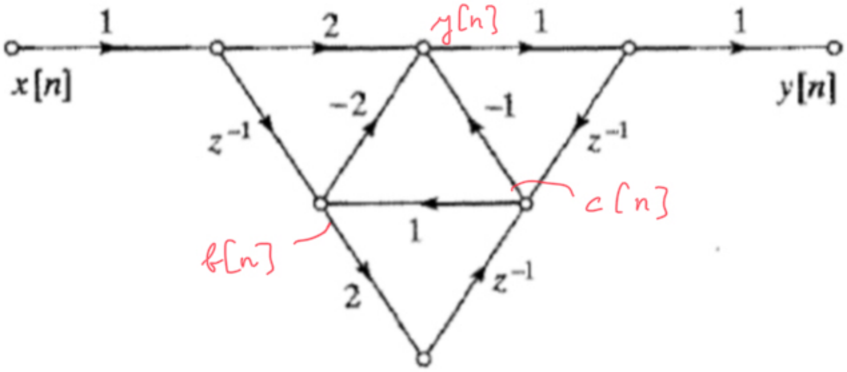



d/




e/ $$H(z) = \frac{2 - 1.5z^{-1} - 0.1z^{-2}}{1 - 0.5z^{-1} - 0.6z^{-2} + 0.9z^{-3}}$$

$$\Rightarrow y[n] = 2x[n] - 1.5x[n-1] - 0.1x[n-2] + 0.5y[n-1] + 0.6y[n-2] - 0.9y[n-3]$$



1
2
y[n]
1
1
x[n]
y[n]
-2
-1
z^-1
z^-1
c[n]
1
b[n]
2
z^-1

(4) $b[n] = x[n-1] + c[n]$

$$c[n] = 2b[n-1] + y[n-1]$$

$$= 2x[n-2] + 2c[n-1] + y[n-1]$$

$$\Rightarrow C(z) = 2z^{-2}X(z) + 2z^{-1}C(z) + z^{-1}Y(z)$$

$$\Rightarrow C(z) = \frac{2z^{-2}X(z) + z^{-1}Y(z)}{1 - 2z^{-1}}$$

$$y[n] = 2x[n] - \overbrace{2b[n]}^{(2x[n-1] + 2c[n])} - c[n]$$

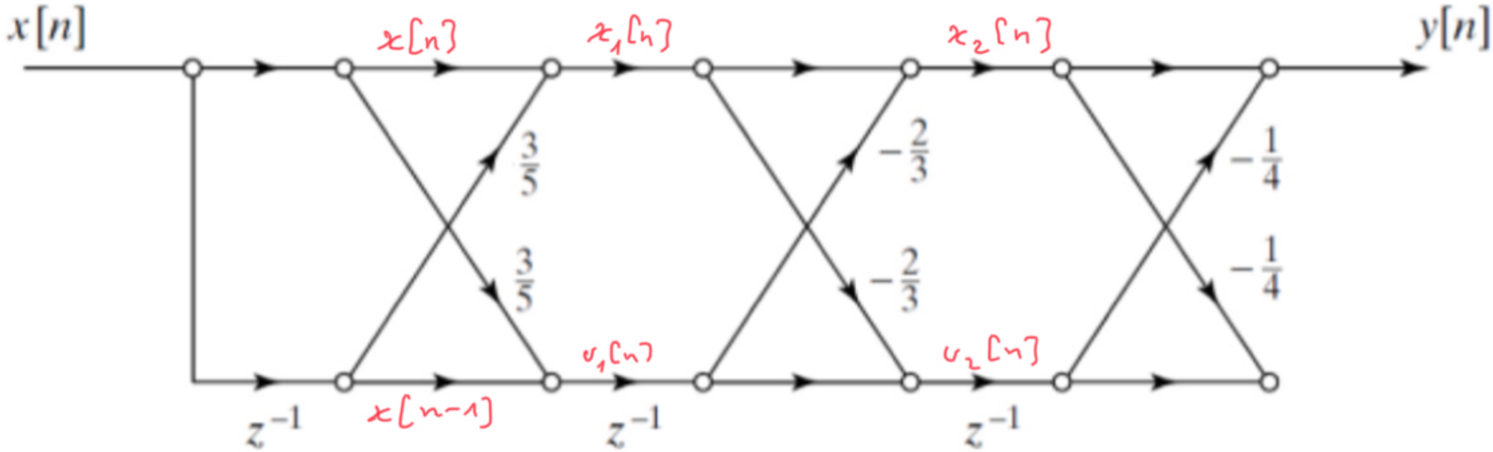
$$= 2x[n] - 2x[n-1] - 3c[n]$$

$$\Rightarrow Y(z) = 2X(z) - 2z^{-1}X(z) - 3C(z)$$

$$= 2X(z) - 2z^{-1}X(z) - 3\frac{2z^{-2}X(z) + z^{-1}Y(z)}{1 - 2z^{-1}}$$

$$\Rightarrow (1 + z^{-1})Y(z) = 2(1 - 3z^{-1} - z^{-2})X(z)$$

$$\Rightarrow y[n] + y[n-1] = 2x[n] - 6x[n-1] - 2x[n-2]$$




(5) $x_1[n] = x[n] + \frac{3}{5} x[n-1]$

$$v_1[n] = x[n-1] + \frac{3}{5} x[n]$$

$$x_2[n] = x_1[n] - \frac{2}{3} v_1[n-1]$$

$$v_2[n] = v_1[n-1] - \frac{2}{3} x_1[n]$$

$$y[n] = x_2[n] - \frac{1}{4} v_2[n-1]$$

$$= \left( x_1[n] - \frac{2}{3} v_1[n-1] \right) - \frac{1}{4}\left( v_1[n-2] - \frac{2}{3} x_1[n-1] \right)$$

$$= \left( x[n] + \frac{3}{5} x[n-1] \right) - \frac{2}{3}\left( x[n-2] + \frac{3}{5} x[n-1] \right)$$
$$- \frac{1}{4}\left( x[n-3] + \frac{3}{5} x[n-2] \right) + \frac{1}{6}\left( x[n-1] + \frac{3}{5} x[n-2] \right)$$

$$= \left( x[n] + \frac{3}{5} x[n-1] \right) - \frac{2}{3}\left( x[n-2] + \frac{3}{5} x[n-1] \right)$$
$$- \frac{1}{4}\left( x[n-3] + \frac{3}{5} x[n-2] \right) + \frac{1}{6}\left( x[n-1] + \frac{3}{5} x[n-2] \right)$$

$$= x[n] + \frac{11}{30} x[n-1] - \frac{43}{60} x[n-2] - \frac{1}{4} x[n-3]$$