ECE 464/564: Digital Signal Processing - Winter 2020
Homework 7

Due: March 3, 2020 (Tuesday)

1. (a) The system function of a discrete-time system is given by

$$H(z) = \frac{10}{1 - e^{0.3}z^{-1}} - \frac{10}{1 - e^{-0.6}z^{-1}}.$$

Assume that this discrete-time filter was designed by the impulse invariance method with $T_d = 3$, i.e., $h[n] = T_d h_c(nT_d)$, where $h_c(t)$ is real. Find the system function $H_c(s)$ of a continuous-time filter that could have been the basis for the design. Plot the frequency response (in terms of magnitude and phase) for both the continuous-time and discrete-time filters using MATLAB.

(b) The system function of a discrete-time system is given by

$$H(z) = \frac{3 + \frac{3}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}.$$

Assume that $H(z)$ was obtained by the bilinear transform method with $T_d = 2$. Find the system function $H_c(s)$ of a continuous-time filter that could have been the basis for the design. Plot the frequency response (in terms of magnitude and phase) for both the continuous-time and discrete-time filters using MATLAB.

(Hint: Use commands `freqs` and `freqz` to get the frequency responses.)

2. Design the system function $H_1(z)$ of the lowest-order Butterworth filter and the system function $H_2(z)$ of the lowest-order Chebyshev Type I filter that meet the following specifications:

$$0.95 \le \left|H(e^{j\omega})\right| \le 1, \quad 0 \le |\omega| \le 0.4\pi,$$
$$\left|H(e^{j\omega})\right| \le 0.1, \quad 0.6\pi \le |\omega| \le \pi.$$

Assume that $T_d = 1$ and aliasing will not be a problem. For each of the filter design (i.e., $H_1(z)$ and $H_2(z)$), answer the following questions:

(a) Sketch the tolerance bounds on the magnitude of the frequency response $|H_c(j\Omega)|$ of the continuous-time filter such that after application of the impulse invariance method, i.e., $h[n] = T_d h_c(nT_d)$, the resulting discrete-time filter will satisfy the given design specifications.

(b) Determine the filter parameters such that the continuous-time filter tightly meets the specifications determined in part (a).

(c) Determine system function of the continuous-time filter and get system function of the discrete-time filter by impulse invariance. (Hint: to simplify the calculation, `zp2tf` function can be used for calculating transfer function, and `residue` function can be used to do partial fraction expansion in MATLAB.)

(d) Plot the magnitude of the resulting discrete-time filter using MATLAB.

3. *Bonus question (equivalent to 1 homework)*: Download the two attached files. A piece of music is added with a high-pass noise. Please design a low-pass filter to eliminate this noise. The specification of that high-pass noise is: $f_{stop} = 10$ kHz and $f_{pass} = 12$ kHz. The original music has a sample rate equals to $f_{sample} = 44.1$ kHz.

   You can use command `sound` in MATLAB to play the music (`sound`(y, f_s) sends audio signal y to the speaker at sample rate f_s). Choose one of the following filter types to design the filter in MATLAB:

   - Chebyshev type I
   - Chebyshev type II
   - Butterworth

   Fill out the attached MATLAB code with calculated design variables and include generated plots.