ECE 464/564: Digital Signal Processing - Winter 2020
Homework 8

Due: March 12, 2020 (Thursday)

1. Suppose that we wish to design an FIR lowpass filter with the following specifications

$$
\begin{aligned}
0.98 \leq \left|H(e^{j\omega})\right| &\leq 1.02, \quad 0 \leq |\omega| \leq 0.5\pi, \\
\left|H(e^{j\omega})\right| &\leq 0.02, \quad 0.6\pi \leq |\omega| \leq \pi.
\end{aligned}
$$

by applying a window to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff frequency $\omega_c = 0.55\pi$.

   (a) For each the following windows: rectangular, Bartlett, Hann, and Hamming, modify the MATLAB code `HW8_prob1_code.m` to determine the minimum value of M that satisfies the specification.

   (b) To support your answer, for each window plot the frequency response of the filter you generated in part (a). Show that with $M-1$ the constraints are not satisfied.

2. An ideal discrete-time Hilbert transformer is a system that introduces $-90°$ ($-\pi/2$ radians) of phase shift for $0 < \omega < \pi$, and $+90°$ ($\pi/2$ radians) of phase shift for $-\pi < \omega < 0$. The magnitude of the frequency response is constant (unity) for $-\pi < \omega < 0$ and for $0 < \omega < \pi$. Such systems are also called ideal $90°$ phase shifters:

$$
H_d(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0. \end{cases}
$$

   (a) Plot the phase response of this system for $-\pi < \omega < \pi$.

   (b) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use $H_d(e^{j\omega})$ given above to determine the FIR impulse response $h[n]$ such that $h[n] = 0$ for $n < 0$ and $n > M$.

   (c) What type(s) of FIR linear-phase systems (I, II, III, or IV) can be used to approximate the ideal Hilbert transformer in part (a)?

3. (a) Consider designing a discrete-time filter with system function $H(z)$ from a continuous-time filter with rational system function $H_c(s)$ by the transformation

$$
H(z) = H_c(s)\Big|_{s=\beta[(1-z^{-\alpha})/(1+z^{-\alpha})]}
$$

   where α is a non-zero integer and β is real. If $\alpha > 0$, for what values of β does a stable, causal continuous-time filter with rational $H_c(s)$ always lead to a stable, causal discrete-time filter with rational $H(z)$?

   (b) A minimum-phase continuous-time system has all its poles and zeros in the left-half s-plane. If a minimum-phase continuous-time system is transformed into a discrete-time system by bilinear transformation, is the resulting discrete-time system always minimum-phase? Explain.