

Q1

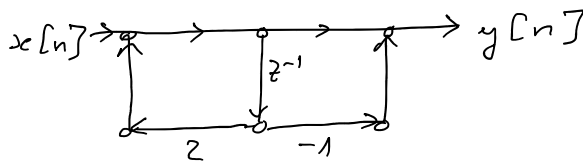
a) From the DF II, we have

$$H(z) = \frac{1 - 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2z^{-2}} = \frac{(1 - z^{-1})^2}{(1 - z^{-1})(1 - 2z^{-1})}$$

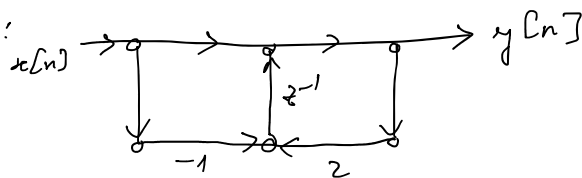
$$= \frac{1 - z^{-1}}{1 - 2z^{-1}} \quad |z| > 2$$

→ (multiplicative form)

b) DF II:



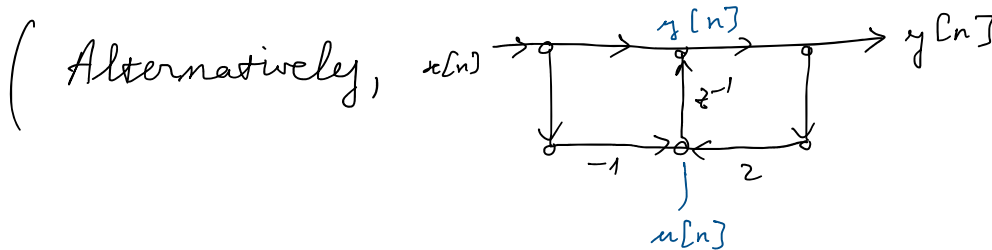
c) Transpose:



d) Transpose should yield the same transfer function.

⇒

$$H(z) = \frac{1 - z^{-1}}{1 - 2z^{-1}}$$



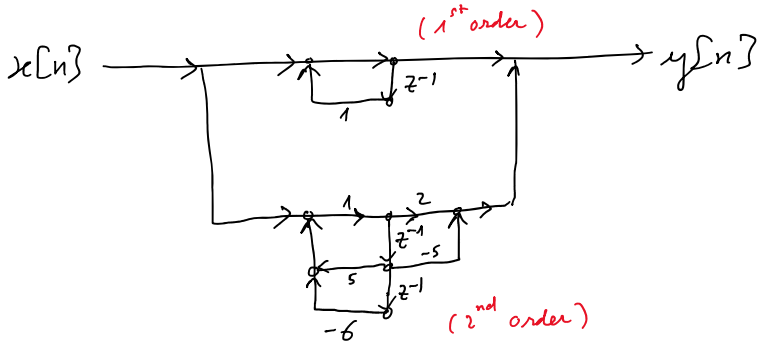
$$\begin{cases} u[n] = -x[n] + 2y[n] \\ y[n] = u[n-1] + x[n] = (-x[n-1] + 2y[n-1]) + x[n] \end{cases}$$

$$\Rightarrow y[n] - 2y[n-1] = x[n] - x[n-1]$$

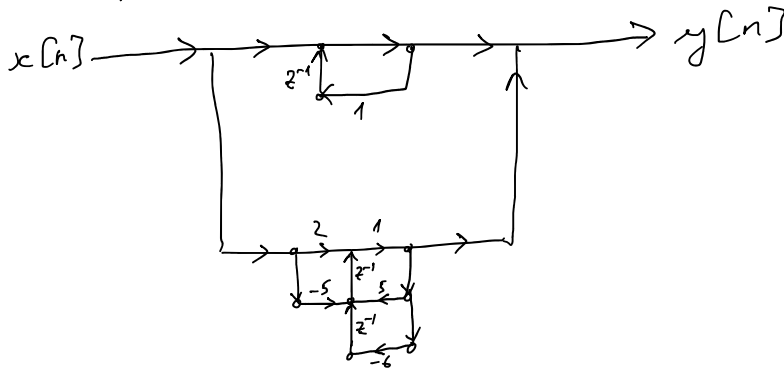
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - 2z^{-1}}$$

Q2

$$a/ \quad H(z) = \frac{1}{1-z^{-1}} + \frac{z-5z^{-1}}{1-5z^{-1}+6z^{-2}}$$

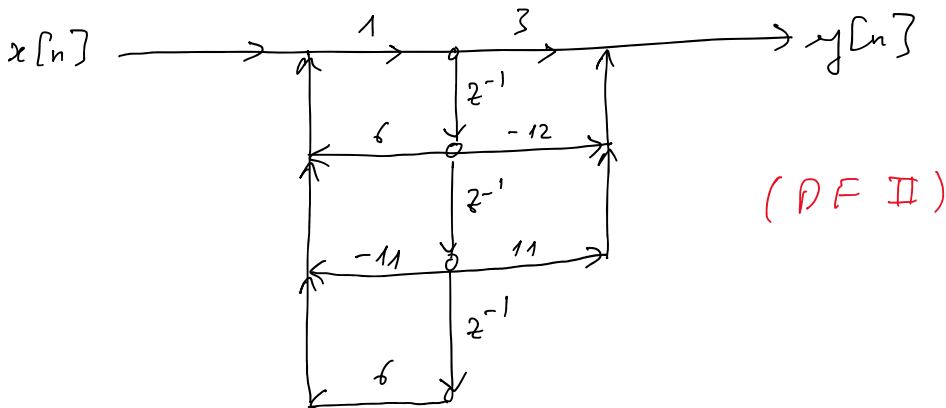


b/ Transpose:



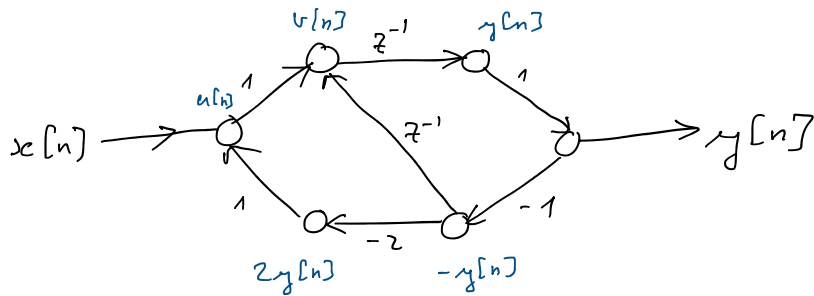
$$c/ \quad H(z) = \frac{(1-5z^{-1}+6z^{-2}) + (z-7z^{-1}+5z^{-2})}{(1-z^{-1})(1-5z^{-1}+6z^{-2})}$$

$$= \frac{z - 12z^{-1} + 11z^{-2}}{1 - 6z^{-1} + 11z^{-2} - 6z^{-3}}$$



Q3

a/



$$\left\{ \begin{array}{l} u[n] = x[n] + 2y[n] \quad (1) \\ v[n] = u[n] - y[n-1] \\ y[n] = v[n-1] \end{array} \right\} \Rightarrow y[n] = u[n-1] - y[n-2] \quad (2)$$

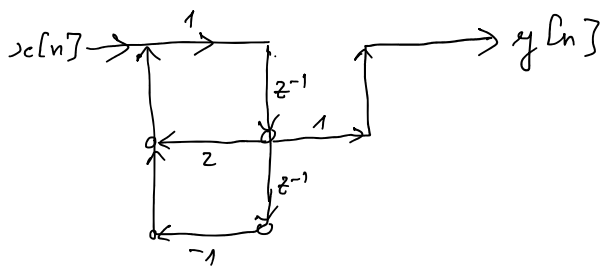
Substituting $u[n-1]$ from (1) into (2) yields:

$$\Rightarrow y[n] = x[n-1] + 2y[n-1] - y[n-2]$$

b/

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

c/



Q4

a/ For a GLP system:

+1 $z = 0.5$ is a zero of $H(z)$

$\Rightarrow z = \frac{1}{0.5} = 2$ is also a zero of $H(z) \Rightarrow a = 2$

+1 $z = j$ is a zero of $H(z)$

$\Rightarrow z = \frac{1}{j} = -j$ is also a zero of $H(z) \Rightarrow b = -j$

+1 $H(e^{j\omega}) = 0$ at $\omega = 0 \Rightarrow z = e^{j0} = 1$ is a zero of $H(z)$

$\Rightarrow c = 1$

b/ Type IV. Since $M = 5$ (odd) and $H(z)$ has one zero at 1, (Also from (c) $\Rightarrow h[n]$ is anti-symmetric)

$$c/ H(z) = (1 - 0.5z^{-1})(1 - jz^{-1})(1 - 2z^{-1})(1 + jz^{-1})(1 - z^{-1})$$

$$= (1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})(1 - jz^{-1})(1 + jz^{-1})(1 - z^{-1})$$

$$= (1 - \frac{5}{2}z^{-1} + z^{-2})(1 + z^{-2})(1 - z^{-1})$$

$$= (1 - \frac{5}{2}z^{-1} + z^{-2})(1 - z^{-1} + z^{-2} - z^{-3})$$

$$= 1 - \frac{5}{2}z^{-1} + z^{-2} - z^{-1} + \frac{5}{2}z^{-2} - z^{-3}$$

$$+ z^{-2} - \frac{5}{2}z^{-3} + z^{-4} - z^{-3} + \frac{5}{2}z^{-4} - z^{-5}$$

$$= 1 - \frac{7}{2}z^{-1} + \frac{9}{2}z^{-2} - \frac{9}{2}z^{-3} + \frac{7}{2}z^{-4} - z^{-5}$$

$$\Rightarrow h[n] = \delta[n] - \frac{7}{2}\delta[n-1] + \frac{9}{2}\delta[n-2] - \frac{9}{2}\delta[n-3] + \frac{7}{2}\delta[n-4] - \delta[n-5]$$