Lecture 1: Introduction and pn Junctions 1 - Junction Formation
Sze And Ng: Chapter 1

Lecture 1
• What are we Going to Cover?
• Course Logistics.
• Regulations.
• pn-Junctions - Basics.
• pn-Junctions - Electrostatics.
What are we Going to Cover in this Course?

Device Physics Sequence
- ECE614 - Semiconductors.
- Last term – Fall term even years.
  - Energy bands [Schrödinger Equation to E vs. k].
  - Semiconductors in equilibrium [charge neutrality].
  - Semiconductors in non-equilibrium [continuity equations].
  - Transport [BTE, continuity equations].
Device Physics Sequence

- ECE615 - Semiconductor Devices I.
- This term – Winter term odd years.
- Two-Terminal Semiconductor Devices.
  - pn junctions.
  - Schottky barriers.
  - MOS (Metal-oxide-semiconductor) capacitors
  - Heterojunctions.
  - Quantum devices.

New for 2021

Device Physics Sequence

- ECE616 - Semiconductor Devices II.
- Next term – Spring term odd years.
- Three-Terminal Semiconductor Devices.
  - Bipolar junction transistors (BJTs).
  - Junction field-effect transistors (JFETs).
  - Metal semiconductor field-effect transistors (MESFETs).
  - Metal oxide semiconductor field-effect transistors (MOSFETs)
  - Heterojunction field-effect transistors (HFETs).
pn Junctions

- What happens when we put a p-doped semiconductor in contact with an n-doped semiconductor?

We cover this subject in extensive detail.
Metal Semiconductor Contacts

- What happens when we put a metal in contact with a semiconductor?

\[ E \quad F \]
\[ n\text{-Type} \quad p\text{-Type} \]

\[ W \]
\[ \phi_B \]

\[ E_F \]

\[ E_F \]

Metal | Semiconductor | Metal | Semiconductor

Emission mechanisms:

Fig. 16 Five basic transport processes under forward bias. (1) Thermionic emission. (2) Diffusion. (3) Recombination. (4) Diffusion of electrons. (5) Diffusion of holes.
MOS Capacitors

- Another important 2-terminal device.

MOS Capacitors

- Capacitance as a function of voltage will be explored extensively.
Heterojunctions

- The interface between two dissimilar semiconductors:

\[ \text{E}_1 \quad \Phi_1 \quad \chi_1 \quad \text{F}_1 \quad \text{C} \]

\[ \text{E}_2 \quad \Phi_2 \quad \chi_2 \quad \text{F}_2 \quad \text{C} \]

Again, we will be calculating band structure and IV characteristics.
Quantum Devices

- We will look at 2-terminal devices that have very small dimensions and/or take advantage of quantum mechanics directly.

Course Logistics
Instructor

- John Labram.
- Assistant Professor, Electrical Engineering and Computer Science.
- Office Location: 3103 Kelley Engineering Center. Online.
- Office Hours: Monday 13:00 – 14:00.
- Email: john.labram@oregonstate.edu.
- Office Hours will be held remotely (a Zoom link will be sent in advance).

Textbook

- Physics of Semiconductor Devices.
- Simon M. Sze and Kwok K. Ng.
- [https://www.amazon.com/Physics-Semiconductor-Devices-Simon-Sze/dp/0471143235/](https://www.amazon.com/Physics-Semiconductor-Devices-Simon-Sze/dp/0471143235/)
- This is a very important book generally (it has received >50,000 times according to Google Scholar).
- I will assign reading from this book.
- But we are only interested in Part II (chapters 2-4).
Lectures

• The lectures are compulsory to watch.
• For Winter 2021, all ECE615 lectures will be delivered remotely.
• Lectures will be delivered synchronously via Zoom.
• A new link will be sent out every Tuesday / Thursday morning via email.
• All lectures will be recorded and uploaded to Canvas later in the day.
  • You can re-watch any lectures / parts of lectures at a later date.

Lectures

• Videos will be linked from the course website as well as on Canvas.
Lectures

• To ask questions please either unmute yourself and speak or type your questions in the chat.
  • I will answer the questions in real time.
  • I will also make sure to repeat the question for the benefit of those not watching live / re-watching.
• The in-person lectures would be compulsory to attend, however I am not implementing any sort of recording of attendance for video lectures.

Lectures

• There will be more equations than my other classes (ECE611, ECE613, ECE418/518, ECE617).
• The teaching is designed to be more inline with the other classes in this series: ECE614, ECE615, ECE616.
• Predominantly PowerPoint based.
  • However, there will be a significant amount of mathematics and derivations.
  • All steps in derivations will be included in the notes, but we may go over some steps reasonably quickly in class.
Lectures

- The slides are designed to be self-contained and to provide enough information to complete all the homeworks / exams.
- However the book will make things easier.
- Some lectures will contain examples, but the book and homeworks will serve as the best opportunity to practice.
- The slides will also be uploaded to the course website at the start of each week.

Course Website

- The course website can be found:
  - http://classes.engr.oregonstate.edu/eecs/winter2021/ece615/.
- Homeworks, solutions, lecture slides, and general information will be located here.
- It should be the first place you look for course information.
Schedule

• This is a preliminary schedule – is subject to change.

<table>
<thead>
<tr>
<th>Week</th>
<th>Day</th>
<th>Date</th>
<th>Month</th>
<th>Lecture #</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tue</td>
<td>5 Jan</td>
<td>Jan</td>
<td>1</td>
<td>Introduction / pn Junctions – Junction Formation</td>
</tr>
<tr>
<td>1</td>
<td>Thur</td>
<td>7 Jan</td>
<td>Jan</td>
<td>2</td>
<td>pn Junctions – Band diagrams</td>
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<tr>
<td>2</td>
<td>Tue</td>
<td>12 Jan</td>
<td>Jan</td>
<td>3</td>
<td>pn Junctions – Band Diagrams cont. and Capacitance</td>
</tr>
<tr>
<td>2</td>
<td>Thur</td>
<td>14 Jan</td>
<td>Jan</td>
<td>4</td>
<td>pn Junctions – Current Voltage Characteristics</td>
</tr>
<tr>
<td>3</td>
<td>Tue</td>
<td>19 Jan</td>
<td>Jan</td>
<td>5</td>
<td>pn Junctions – Non-ideal characteristics</td>
</tr>
<tr>
<td>3</td>
<td>Thur</td>
<td>21 Jan</td>
<td>Jan</td>
<td>6</td>
<td>pn Junctions – Transient Behavior</td>
</tr>
<tr>
<td>4</td>
<td>Tue</td>
<td>26 Jan</td>
<td>Jan</td>
<td>7</td>
<td>Navigating Device Literature</td>
</tr>
<tr>
<td>4</td>
<td>Thur</td>
<td>28 Jan</td>
<td>Jan</td>
<td>8</td>
<td>Schottky Contacts – Metal Semiconductor Contacts</td>
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<tr>
<td>5</td>
<td>Tue</td>
<td>2 Feb</td>
<td>Feb</td>
<td>9</td>
<td>Schottky Contacts – Depletion Region Issues</td>
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<tr>
<td>5</td>
<td>Thur</td>
<td>4 Feb</td>
<td>Feb</td>
<td>10</td>
<td>Schottky Contacts – IV Characteristics</td>
</tr>
<tr>
<td>6</td>
<td>Tue</td>
<td>9 Feb</td>
<td>Feb</td>
<td>11</td>
<td>MOS Capacitors – Energy Band Diagrams</td>
</tr>
<tr>
<td>6</td>
<td>Thur</td>
<td>11 Feb</td>
<td>Feb</td>
<td>12</td>
<td>MOS Capacitors – Ideal electrostatics</td>
</tr>
<tr>
<td>7</td>
<td>Tue</td>
<td>16 Feb</td>
<td>Feb</td>
<td>13</td>
<td>MOS Capacitors – Non-ideal electrostatics</td>
</tr>
<tr>
<td>7</td>
<td>Thur</td>
<td>18 Feb</td>
<td>Feb</td>
<td>14</td>
<td>Heterojunctions – Classification / Energy bands</td>
</tr>
<tr>
<td>8</td>
<td>Tue</td>
<td>23 Feb</td>
<td>Feb</td>
<td>15</td>
<td>Heterojunctions – Electrostatics</td>
</tr>
<tr>
<td>8</td>
<td>Thur</td>
<td>25 Feb</td>
<td>Feb</td>
<td>16</td>
<td>Heterojunctions – IV Characteristics</td>
</tr>
<tr>
<td>9</td>
<td>Tue</td>
<td>28 Feb</td>
<td>Feb</td>
<td>17</td>
<td>Quantum Devices - Quantum Mechanics</td>
</tr>
<tr>
<td>9</td>
<td>Thur</td>
<td>2 Mar</td>
<td>Mar</td>
<td>18</td>
<td>Quantum Devices - 2D and Ballistic Transport</td>
</tr>
<tr>
<td>10</td>
<td>Tue</td>
<td>9 Mar</td>
<td>Mar</td>
<td>19</td>
<td>Quantum Devices - Tunneling Devices</td>
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<tr>
<td>10</td>
<td>Thur</td>
<td>11 Mar</td>
<td>Mar</td>
<td>20</td>
<td>No Lecture</td>
</tr>
</tbody>
</table>

Assessment

• The final grade will consist of the following contributions:

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Percentage of Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>60</td>
</tr>
<tr>
<td>Term Paper 1</td>
<td>20</td>
</tr>
<tr>
<td>Term Paper 2</td>
<td>20</td>
</tr>
</tbody>
</table>

• In normal years there would be a midterm and final exam.
• This year however, there will be two term papers (based on 2 terminal devices in the literature).
Homework

- There will be a total of 4 homeworks.
- Each homework carries equal weight.
- The homeworks are designed to test your understanding of the concepts covered in the lectures.
  - Sometimes you will be expected to apply knowledge obtained in the lectures to new (previously unseen) situations.
- The homeworks overall contribute 60% of the course grade.
  - 15% each.

Homework

- Some homework will be analytical (answerable with pen and paper).
- Some will require simple computation.
  - E.g. I will provide some example data and expect you to carry out numerical integration or similar.
  - Everything you will be asked to do you will be able to do in Excel.
  - Although you can use a programming language if you prefer.
  - You may be expected to extract a parameter or plot a graph for example.
Homework

• Homework will be set on Thursdays, and due 1 week later at 8:30 am.
• Please send an electronic copy of the homework to me (john.labram@oregonstate.edu) on the due date.
  • Can be a Word document, pdf, or series of scanned images.
  • Please make the document as easy as possible to follow. E.g. circle / make a box around final answer.
  • If you scan / photograph your homework please ensure the resolution is high!
• Please use your OSU address (not Gmail etc.)

Homework

• Late homework will be deducted 10% per day late for a maximum of 5 days, after which the homework grade will be zero.
  • For example, if you scored 85% on a homework, but you hand it in 2 days late, you will receive a grade of 65%.
• The homeworks will be returned to you one week after they are submitted (via email).
• The solutions will be posted when the homeworks are returned.
Homework

• The homework schedule is as follows.

<table>
<thead>
<tr>
<th>Homework #</th>
<th>Set</th>
<th>Due</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Week 2: Jan. 14th 2021</td>
<td>Week 3: Jan. 21st 2021</td>
</tr>
<tr>
<td>2</td>
<td>Week 3: Jan. 21st 2021</td>
<td>Week 4: Jan. 28th 2021</td>
</tr>
<tr>
<td>3</td>
<td>Week 6: Feb. 11th 2021</td>
<td>Week 7: Feb. 18th 2021</td>
</tr>
</tbody>
</table>

• Again, this is subject to change.

Literature Reports

• This year, ECE615 will also test your ability to search and extract information from the literature.
• This is because you much of the information in the latter half of the course will be predominantly from research articles, not textbooks.
• Being able to effectively search the literature is a skill in itself that requires practice.
• Lecture 7 will be dedicated to techniques for searching the literature.
Literature Reports

• We will be using Clarivate Web of Science

More detailed instructions will be provided nearer the time.

• You will be expected to complete two Literature Reports on the 2-terminal device literature.

• You can consider these as a hybrid between homework and a term paper.

• It will be a combination of:
  • Short questions (a sentence or two). E.g., “Provide the reference for the first Schottky Diode”
  • Longer questions (1/2 to 1 page). E.g. “Briefly describe reliability issues in resonant tunneling diodes”
Literature Reports

- Each report will contribute 20% to the overall course grade.
- The schedule for these reports is as below:

<table>
<thead>
<tr>
<th>Report #</th>
<th>Set</th>
<th>Due</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Week 4: Jan. 28\textsuperscript{th} 2021</td>
<td>Week 6: Feb. 11\textsuperscript{th} 2021</td>
</tr>
<tr>
<td>2</td>
<td>Week 9: Mar. 4\textsuperscript{th} 2021</td>
<td>Finals Weeks: Mar 16\textsuperscript{th} 2021</td>
</tr>
</tbody>
</table>

- Again, this is subject to change.

Grade Boundaries

<table>
<thead>
<tr>
<th>Lower Bound (%)</th>
<th>Upper Bound (%)</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
<td>100</td>
<td>A</td>
</tr>
<tr>
<td>90</td>
<td>92</td>
<td>A-</td>
</tr>
<tr>
<td>87</td>
<td>89</td>
<td>B+</td>
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<tr>
<td>83</td>
<td>86</td>
<td>B</td>
</tr>
<tr>
<td>80</td>
<td>82</td>
<td>B-</td>
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<tr>
<td>77</td>
<td>79</td>
<td>C+</td>
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<td>73</td>
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<tr>
<td>70</td>
<td>72</td>
<td>C-</td>
</tr>
<tr>
<td>67</td>
<td>69</td>
<td>D+</td>
</tr>
<tr>
<td>63</td>
<td>66</td>
<td>D</td>
</tr>
</tbody>
</table>

- Percentages will be rounded-off to the nearest whole percent to determine letter grade.
Grading

- Hopefully I will not have to curve.
- But it depends on results.
- I may try to ~match grade distributions from previous years.
- But depends on performance, since this is a small class.

Regulations
Cheating and Student Conduct

- Academic dishonesty is defined as an intentional act of deception in one of the following areas:
  - **Cheating** - use or attempted use of unauthorized materials, information or study aids.
  - **Fabrication** - falsification or invention of any information.
  - **Assisting** - helping another commit an act of academic dishonesty.
  - **Tampering** - helping another commit an act of academic dishonesty.
  - **Plagiarism** - representing the words or ideas of another person as one's own.

When evidence of academic dishonesty comes to the instructor's attention, the instructor will document the incident, permit the accused student to provide an explanation, advise the student of possible penalties, and take action.

The instructor may impose any academic penalty up to and including an "F" grade in the course after consulting with his or her department chair and informing the student of the action taken.
Disruptive Behavior

• While the University is a place where the free exchange of ideas and concepts allows for debate and disagreement, all classroom behavior and discourse should reflect the values of respect and civility.
• Behaviors which are disruptive to the learning environment will not be tolerated.
• As your instructors, we are dedicated to establishing a learning environment that promotes diversity of race, culture, gender, sexual orientation, and physical disability.

Disruptive Behavior

• Anyone noticing discriminatory behavior in this class, or feeling discriminated against should bring it to the attention of the instructors or other University personnel as appropriate.
pn-Junctions

pn-Junction Diodes

- While there are various structures that lead to rectification (e.g. Schottky diodes, tunneling diodes), pn-junctions are the most common.
- A pn-junction diode is formed by placing a p-doped semiconductor next to an n-doped semiconductor.
pn-Junction Diodes

- Conceptually this is an easy way to think about pn-junctions.
- And can actually be achieved with epitaxal growth of one semiconductor on top of another.
- Normally however, a pn-junction would be created by adjusting doping properties of a single semiconductor (e.g. Si).
- We will not concern ourselves with this detail today.

Doping

- First, let us remind ourselves what we are doing when we dope a semiconductor.
- N-type:
  - We are adding an electron and a proton in a certain position.
- P-type:
  - We are removing an electron and a proton in a certain position.
Doping

- Hence we are not changing the charge on the atom.
- **A doped semiconductor is neutral!**
- N-type:
  - Because the valence has changed, the extra electron does not participate in bonding.
  - It is very weakly bound to its host atom, and essentially is free to move around the lattice.

\[
\begin{array}{cccc}
Si & Si & Si & Si \\
\text{P} & & & \\
Si & Si & Si & Si \\
\text{N} & & & \\
Si & Si & Si & Si \\
\end{array}
\]

Doping

- Hence we are not changing the charge on the atom.
- **A doped semiconductor is neutral!**
- P-type:
  - Because the valence has changed, there is a broken bond and a missing electron.
  - This absence of an electron (hole) can “move” by electrons around it filling its place.

\[
\begin{array}{cccc}
Si & Si & Si & Si \\
\text{B} & & & \\
Si & Si & Si & Si \\
\text{N} & & & \\
Si & Si & Si & Si \\
\end{array}
\]
Doped Semiconductors

- What does this look like in a simple band diagram?
- Our intrinsic (undoped) semiconductor can be pictured like this:

\[ \text{Conduction band minimum energy} \quad \text{Band Gap} \quad \text{Fermi Energy} \]

- At room-temperature and in the dark, there should be very few free carriers.
- Intrinsic silicon is highly resistive.

Doped Semiconductors

- Doping the semiconductor will, at room temperature, add electrons to the conduction band (n-type) or add holes to the valence band (p-type):
pn-Junctions

- What happens when we bring these two semiconductors into contact?
- Fermi level must be uniform throughout!
  - If not, carriers will move to correct it.

```latex
\begin{align*}
\text{p-type} & & \text{n-type} \\
\begin{array}{ccc}
\text{p} & \text{p} & \text{p} \\
\text{e} & \text{e} & \text{e} \\
\end{array} & & \\
\begin{array}{ccc}
\text{e} & \text{e} & \text{e} \\
\text{e} & \text{e} & \text{e} \\
\end{array} \\
\begin{array}{ccc}
\text{e} & \text{e} & \text{e} \\
\text{e} & \text{e} & \text{e} \\
\end{array} \\
\end{align*}
```

ECE 615 – Semiconductor Devices I
Winter 2021 - John Labram

pn-Junctions

- Carriers are now free to diffuse into adjacent semiconductor.
- We now have holes in the n-type semiconductor and electrons in the p-type semiconductor.

```latex
\begin{align*}
\text{p-type} & & \text{n-type} \\
\begin{array}{ccc}
\text{p} & \text{p} & \text{p} \\
\text{e} & \text{e} & \text{e} \\
\end{array} & & \\
\begin{array}{ccc}
\text{e} & \text{e} & \text{e} \\
\text{e} & \text{e} & \text{e} \\
\end{array} \\
\begin{array}{ccc}
\text{e} & \text{e} & \text{e} \\
\text{e} & \text{e} & \text{e} \\
\end{array} \\
\end{align*}
```

ECE 615 – Semiconductor Devices I
Winter 2021 - John Labram
pn-Junctions

- This will lead to annihilation.
- The region close to the junction will be devoid of free carriers.
  - This is called the **depletion region**.

  ![Diagram of p-type and n-type regions](image)

- Away from the depletion region there are still free carriers.
- We define the depletion width on the n-side $W_{dn}$ and the depletion width on the p-side $W_{dp}$.

  ![Diagram showing depletion widths](image)
pn-Junctions

- What about charge neutrality?
- Away from the interface: \( n \approx N_D \) and \( p \approx N_A \).
- Close to the interface however, we are left with ionized dopant atoms.

Rectification

- We will show the Fermi level must be constant.
- This means the absolute values of the bands must be offset relative to each other.
**Reverse Bias**

- Apply bias from right to left.
- I.e. we are attracting electrons from left to right.

**Carriers** will not be able to continue to flow.
Forward Bias

- Apply bias from left to right.
- I.e. we are attracting electrons from right to left
- As long as we can overcome the barrier potential (e.g. 0.7 eV), carriers can move towards each other.
- Depletion region will shrink.
- Carriers will recombine and can continue to be injected.
Summary of pn Junctions

- At the junction, there is a large gradient of e⁻ and h⁺.
- Diffusion currents flow because of the concentration gradient.
- As the mobile carriers diffuse across the junction and recombine, they leave behind "uncovered" immobile charge (ionized donors and acceptors).
- The uncovered ionized dopants give rise to a built-in electric field which produces a drift current that opposes and eventually exactly balances (equilibrium) the diffusion of mobile carriers.

This region of uncovered immobile charges near the junction is known as the "space charge" region (SCR). It is also known as the "depletion" region due to the depletion of mobile charge carriers.
Electrostatics

Drift Current

- Equilibrium is reached when the overall e⁻ and h⁺ currents (due to drift and diffusion) are equal to zero.
- Let’s start with holes.
- We describe drift current by:

\[ J_p = e \mu_p p \varepsilon \]

- \( J_p \) is hole current density.
- \( e \) is the magnitude of the fundamental unit of charge.
- \( \mu_p \) is the mobility of holes.
- \( p \) is the hole number density.
- \( \varepsilon \) is the electric field strength.
**Diffusion Current**

- Equilibrium is reached when the overall $e^-$ and $h^+$ currents (due to drift and diffusion) are equal to zero.
- Let’s start with holes.
- We describe diffusion current by:

$$J_p = eD_p \frac{dp}{dx}$$

- $J_p$ is hole current density.
- $e$ is the magnitude of the fundamental unit of charge.
- $D_p$ is the diffusion coefficient of holes.
- $p$ is the hole number density.
- $x$ is the diffusion direction.

**Boltzmann Distribution**

- We approximate the distribution of carriers by the Boltzmann Distribution:

$$p = p_i \exp\left(\frac{(E_i - E_F)}{k_B T}\right)$$

- $p$ is hole number density.
- $p_i$ is intrinsic hole number density.
- $E$ is the energy of interest (e.g., conduction band edge).
- $E_F$ is the Fermi energy.
- $k_B$ is the Boltzmann Constant.
- $T$ is the temperature.
Boltzmann Distribution

- The Boltzmann Distribution is an approximation of the Fermi Function:
  \[
  p \approx p_i \exp\left(\frac{E - E_F}{k_B T}\right)
  \]

- Only valid if \( (E - E_F) \gtrsim 3k_B T \).

Electrostatic Potential

- Equilibrium is reached when the overall e\(^-\) and h\(^+\) currents (due to drift and diffusion) are equal to zero.
  \[
  J_P = e\mu_p p\varepsilon - eD_p \frac{dp}{dx} = 0
  \]  \tag{1}

- Describe carriers by Maxwell-Boltzmann distribution:
  \[
  p = p_i \exp\left(\frac{E - E_F}{k_B T}\right)
  \]  \tag{2}

- We need first derivative with respect to \( x \):
  \[
  \frac{dp}{dx} = ?
  \]

Remember: \( p \) is hole density, not momentum here.
Carrier Density Gradient

- Where is the \( x \)-dependence in \( p \)?

\[
p = p_i \exp \left( \frac{E - E_F}{k_B T} \right)
\]

\[E_v\]

\[E_C\]

\[E_F\]

- From our knowledge of the band diagram, we know the energy of a particular state will depend on \( x \).

Carrier Density Gradient

- So we can say:

\[E = E(x)\]

- Even though we have shown that \( E_F \neq E_F(x) \) from intuition, we here want to prove it mathematically.

- So, for now, we will also say:

\[E_F = E_F(x)\]

- I.e.

\[p(x) = p_i \exp \left( \frac{(E(x) - E_F(x))}{k_B T} \right)\]
Carrier Density Gradient

\[ p(x) = p_i \exp \left( \frac{E(x) - E_F(x)}{k_B T} \right) \]

- So, let’s re-write this as:

\[ p(x) = p_i \exp \left( \frac{E(x)}{k_B T} \right) \exp \left( -\frac{E_F(x)}{k_B T} \right) \]  \hspace{1cm} (2a)

- We will hence have to use the product rule:

\[ \frac{d}{dx} (f(x)g(x)) = \frac{df}{dx} g(x) + \frac{dg}{dx} f(x) \]

- And the chain rule:

\[ \frac{dy}{dx} [y(u(x))] = \frac{dy}{du} \frac{du}{dx} \]

Carrier Density Gradient

\[ p(x) = p_i \exp \left( \frac{E(x)}{k_B T} \right) \exp \left( -\frac{E_F(x)}{k_B T} \right) \]  \hspace{1cm} (2a)

- Start with first term. Let:

\[ y(x) = p_i \exp \left( \frac{E(x)}{k_B T} \right) \]

- Apply chain rule:

\[ \frac{dy}{dx} = \frac{dy}{dE} \frac{dE}{dx} \]

\[ \frac{dy}{dE} = \frac{p_i}{k_B T} \exp \left( \frac{E(x)}{k_B T} \right) \]

\[ \frac{dy}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x)}{k_B T} \right) \frac{dE}{dx} \]

\[ \frac{dy}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x)}{k_B T} \right) \frac{dE}{dx} \]
Carrier Density Gradient

\[ p(x) = p_i \exp \left( \frac{E(x)}{k_B T} \right) \exp \left( \frac{-E_F(x)}{k_B T} \right) \]  
(2a)

- Now look at second term:
  \[ z(x) = \exp \left( \frac{-E_F(x)}{k_B T} \right) \]
- Apply chain rule again:
  \[
  \frac{dz}{dx} = \frac{dz}{dE_F} \frac{dE_F}{dx} = \frac{dz}{dE_F} = -\frac{1}{k_B T} \exp \left( \frac{-E_F(x)}{k_B T} \right) \\
  \frac{dz}{dx} = -\frac{1}{k_B T} \exp \left( \frac{-E_F(x)}{k_B T} \right) \frac{dE_F}{dx}
  
\]

And we have said:

\[ p(x) = y(x)z(x) \]

- So far we have:
  \[
  y(x) = p_i \exp \left( \frac{E(x)}{k_B T} \right) \\
  z(x) = \exp \left( \frac{-E_F(x)}{k_B T} \right) \\
  \frac{dy}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x)}{k_B T} \right) \frac{dE}{dx} \\
  \frac{dz}{dx} = -\frac{1}{k_B T} \exp \left( \frac{-E_F(x)}{k_B T} \right) \frac{dE_F}{dx}
  
\]
Carrier Density Gradient

\[ p(x) = y(x)z(x) \]

- We can now use the product rule:

\[
\frac{dp}{dx} = \frac{d}{dx}(y(x)z(x)) = \frac{dy}{dx}z(x) + \frac{dz}{dx}y(x)
\]

\[
\frac{dp}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x) - E_F(x)}{k_B T} \right) \frac{dE}{dx} - \frac{p_i}{k_B T} \exp \left( \frac{E(x) - E_F(x)}{k_B T} \right) \frac{dE_F}{dx}
\]

- We can tidy this up:

\[
\frac{dp}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x) - E_F(x)}{k_B T} \right) \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right]
\]

Carrier Density Gradient

\[
\frac{dp}{dx} = \frac{p_i}{k_B T} \exp \left( \frac{E(x) - E_F(x)}{k_B T} \right) \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right]
\]

- Identify our equation for carrier density (Eqn. 2):

\[ p = p_i \exp \left( \frac{(E - E_F)}{k_B T} \right) \]

- Hence we can say:

\[
\frac{dp}{dx} = \frac{p}{k_B T} \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right]
\]
**Electric Field Strength**

\[ \frac{dp}{dx} = \frac{p}{k_B T} \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right] \]  

- From the definition of voltage \((V)\):
  \[ \mathcal{E} = \frac{dV}{dx} \]
- Voltage and energy are related by:
  \[ E = eV \]
- Hence we can say:
  \[ \frac{dE}{dx} = e\mathcal{E} \]  

**Electrostatic Potential**

- We also need to use the Einstein relationship.
  \[ D_p = \frac{k_B T}{e} \mu_p \]  
  \[ (5) \]
- (i.e. more mobile / hotter charge carriers will diffuse more readily).
- Recall our equation for current density:
  \[ J_p = e\mu_p p\mathcal{E} - eD_p \frac{dp}{dx} = 0 \]  
  \[ (1) \]
- Substitute (5) into (1):
  \[ J_p = e\mu_p p\mathcal{E} - e\frac{k_B T}{e} \mu_p \frac{dp}{dx} = 0 \]
Electrostatic Potential

\[ J_p = e\mu_p p\varepsilon - k_B T \mu_p \frac{dp}{dx} = 0 \]

Now we can substitute in our equation for carrier density gradient (Equation 3):

\[
\frac{dp}{dx} = \frac{p}{k_B T} \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right]
\]

\[ J_p = e\mu_p p\varepsilon - k_B T \mu_p \frac{p}{k_B T} \left[ \frac{dE}{dx} - \frac{dE_F}{dx} \right] = 0 \]

\[ J_p = \mu_p p \left( e\varepsilon - \frac{dE}{dx} + \frac{dE_F}{dx} \right) = 0 \]

Finally, use our equation for energy gradient:

\[ \frac{dE}{dx} = e\varepsilon \]  

Substitute into the current density:

\[ J_p = \mu_p p \left( \frac{dE}{dx} - \frac{dE}{dx} + \frac{dE_F}{dx} \right) = 0 \]

\[ J_p = \mu_p p \frac{dE_F}{dx} = 0 \]

Since mobility and density are not generally zero, we say:

\[ \frac{dE_F}{dx} = 0 \]
Summary

- We have introduced the course and have begun looking at the formation of pn junctions.

Next Time

- We will look at pn junction band diagrams.