Lecture 5:

pn Junctions – Non-Ideal Current-Voltage Characteristics

Sze And Ng: Chapter 2

Announcements

Homework 1/4:
• Is online now.
• Total of 25 marks.
• Due Thursday 21st January at the start of the lecture (08:30am).
  • Email it to me at john.labram@oregonstate.edu.
• I will return it one week later (January 28th).
• Homework 1 consists of content covered in Lectures 1, 2, 3, 4.
• You can download the homework from the Canvas or the course website:
  • http://classes.engr.oregonstate.edu/eecs/winter2021/ece615/homework.html
Announcements

Homework 2/4:

- Will be online after the next lecture (Thursday 21st January).
- Due **Thursday 28th January at the start of the lecture (08:30am).**
- Email it to me at john.labram@oregonstate.edu.
- I will return it one week later (February 4th).
- Homework 2 consists of content covered in Lectures 5, 6.

Last Time

- We spent last lecture deriving the Ideal Diode Equation (the Shockley Equation):
  \[ J = J_0 \left[ \exp \left( \frac{eV}{k_BT} \right) - 1 \right] \]
  \[ J_0 = \frac{eD_p p_0}{L_p} + \frac{eD_n n_0}{L_n} = \frac{eD_p n_i^2}{L_p N_D} + \frac{eD_n n_i^2}{L_n N_A} \]
Lecture 5

- Recombination-Generation Current.
- High-Level Injection and Series Resistance.
- Threshold Voltage.
- Breakdown.

Recombination-Generation Current
Ideal I-V Characteristics

- Last time we derived the ideal diode or Shockley Equation that quantitatively describes the flow of current through a biased pn junction.

\[ J = J_0 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right] \]  

(1)

- Describes the current that flows in a diode as a function of voltages applied to it.
- Where we define the reverse saturation current:

\[ J_0 = \frac{eD_p p_0}{L_p} + \frac{eD_n n_0}{L_n} \]  

(2)

Non-Ideal IV

- We made a lot of assumptions in the last lecture.
- In reality there are many generalizations we need to make to accurately predict actual device behavior.
Non-Ideal IV

- Actual diode current deviates from our simple ideal model in a number of significant ways:
  1. Additional forward current due to recombination in depletion region.
  2. Additional reverse leakage due to generation in depletion region.
  3. High injection and series resistance effects.
  4. Threshold Voltage.
  5. Breakdown.

Recombination-Generation Current

- Last time we only mentioned the net recombination rate ($U$) very briefly.
- We simply described it via the minority carrier lifetime ($\tau_p$):
  \[ \tau_p = \frac{p_n - p_{n0}}{U} \]  
  (3)
- Where:
  - $p_n$ is the hole concentration on the $n$ side.
  - $p_{n0}$ is the hole concentration on the $n$ side in equilibrium (zero external bias).
SRH Recombination

- We can describe this recombination as Shockley-Read-Hall (SRH) recombination.
- Also often called monomolecular or trap-assisted recombination.
- SRH is basically due to the presence of trap states in the band gap.
- $E_t$ is the energy of this trap state.
- $N_t$ is the spatial number density of trap states.

Recombination-Generation Current

- Another important current mechanism involves generation and recombination in the space charge region (SCR).
- According to Shockley-Read-Hall (SRH) theory, the net rate of recombination is given by the expression:

$$U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n \left[ n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right) \right] + \sigma_p \left[ p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right) \right]}$$

- See Eqn. 92, Chapter 1 Sze & Ng.
- Variables labelled on next slides.
Capture Cross Sections

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n [n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right)] + \sigma_p [p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right)]} \]

- \( \sigma_n \) is the electron capture cross section.
- \( \sigma_p \) is the hole capture cross section.

\( v_{th} \) is the carrier thermal velocity.
\( N_t \) is the trap site density.
\( p \) is hole density.
\( n \) is electron density.
\( n_i \) is the intrinsic carrier concentration.
\( E_t \) is the trap energy.
\( E_i \) is the energy we are interested in.
\( k_B \) is the Boltzmann Constant.
\( T \) is Temperature.

Recombination-Generation Current
Recombination-Generation Current

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (p n - n_i^2)}{\sigma_n \left[ n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right) \right] + \sigma_p \left[ p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right) \right]} \]

- A couple of things worth mentioning:
  - \( U \) has dimensions of \( L^{-3} T^{-1} \). E.g. \( \text{cm}^{-3} \text{s}^{-1} \).
  - This is not just a rate (c.f. \( \tau^{-1} \) or \( k_1 \)).
  - What if \( p n = n_i^2 \)? I.e. we are at equilibrium.
    - \( U = 0 \).
    - Hence if \( U \neq 0 \), we have disturbed equilibrium.

Recombination-Generation Current

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (p n - n_i^2)}{\sigma_n \left[ n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right) \right] + \sigma_p \left[ p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right) \right]} \]

- What does the sign of \( U \) mean?
  - If \( p n > n_i^2 \), \( U > 0 \): Recombination.
  - If \( p n < n_i^2 \), \( U > 0 \): Generation.
  - This comes from the driving force to restore equilibrium.
  - \( U \propto \sigma_n, U \propto \sigma_p, U \propto N_t, U \propto v_{th} \).
  - I.e. the magnitude of the rate will increase with each of these variables.
Energy Spectrum

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n \left[ n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right) \right] + \sigma_p \left[ p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right) \right]} \]

- What about \( E_t \)?
  - \( E_i \) is the Fermi Energy of the intrinsic semiconductor (before we dope it).
  - This tells us there will be an energy spectrum for the rate \( U \) depending on trap energy.
  - We can show that \( U \) is maximized for \( E_i = E_t \).
  - The traps closest to the center are most efficient.

Simplifications

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n \left[ n + n_i \exp \left( \frac{E_t - E_i}{k_B T} \right) \right] + \sigma_p \left[ p + n_i \exp \left( \frac{E_i - E_t}{k_B T} \right) \right]} \]

- To make further progress we need a few simplifications.
- Let’s start by considering only the most influential traps: those that are in the middle of the gap.
  \[ E_t = E_i \] (5)
- In this case, we find:
  \[ U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n [n + n_i] + \sigma_p [p + n_i]} \] (6)
Simplifications

\[ U = \frac{\sigma_n \sigma_p v_{th} N_t (pn - n_i^2)}{\sigma_n [n + n_i] + \sigma_p [p + n_i]} \]  

(6)

- Last time, we talked about Fermi-level splitting when you apply a voltage:

\[
\begin{align*}
\text{Forward Bias} & & \text{Reverse Bias} \\
E_F = & E_F & E_F = & E_F \\
E_C = & E_C & E_C = & E_C \\
-E_{Fp} = & -E_{Fp} & -E_{Fp} = & -E_{Fp} \\
-E_{Fp} = & -E_{Fp} & -E_{Fp} = & -E_{Fp} \\
& x & & x \\
\end{align*}
\]

Fermi-Level Splitting

- We also showed last time that:

\[ pn = n_i^2 \exp \left( \frac{E_{Fn} - E_{Fp}}{k_B T} \right) \]  

(7)

- By definition, the splitting of the Fermi Levels is due to the applied voltage. Hence we can say:

\[ eV = E_{Fn} - E_{Fp} \]  

(8)

- Substituting into (7):

\[ pn = n_i^2 \exp \left( \frac{eV}{k_B T} \right) \]  

(9)
Simplifications

- Putting (9) into our equation for $U$:

$$U = \frac{\sigma_n \sigma_p \nu_{th} N_t n_i^2 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{\sigma_n [n + n_i] + \sigma_p [p + n_i]}$$  \hspace{1cm} (10)

- Finally, we assume the cross-sections for holes and electrons are equal.

$$\sigma_n = \sigma_p \equiv \sigma$$  \hspace{1cm} (11)

- Giving:

$$U = \frac{\sigma \nu_{th} N_t n_i^2 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{n + p + 2n_i}$$  \hspace{1cm} (12)

- Recall from last lecture we can describe the carrier concentrations in terms of the quasi-Fermi Levels:

$$n = n_i \exp \left( \frac{E_{F_n} - E_i}{k_B T} \right)$$ \hspace{1cm} (13)

$$p = n_i \exp \left( \frac{E_i - E_{F_p}}{k_B T} \right)$$ \hspace{1cm} (14)

- Hence we can say:

$$U = \frac{\sigma \nu_{th} N_t n_i^2 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{n_i \left( \exp \left( \frac{E_{F_n} - E_i}{k_B T} \right) + \exp \left( \frac{E_i - E_{F_p}}{k_B T} \right) + 2 \right)}$$  \hspace{1cm} (15)
Simplifications

- Recall, we are interested in traps where $E_t = E_i$.
- I.e. traps in the center of the gap in equilibrium.
- With an applied bias $E_i$ will be halfway between $E_{Fn}$ and $E_{Fp}$.

Forward Bias

Reverse Bias

\[ U = \frac{\sigma v_{th} N_t n_i^2 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{n_i \left( \exp \left( \frac{2E_{Fn} - 2E_i}{2k_B T} \right) + \exp \left( \frac{2E_i - 2E_{Fp}}{2k_B T} \right) + 2 \right) + \exp \left( \frac{2E_{Fn} - 2E_i - E_{Fp}}{2k_B T} \right) + \exp \left( \frac{E_{Fn} + E_{Fp} - 2E_{Fp}}{2k_B T} \right) + 2} \]

If $E_i$ is halfway between $E_{Fn}$ and $E_{Fp}$:

\[ E_i = \frac{E_{Fn} + E_{Fp}}{2} \]

\[ 2E_i = E_{Fn} + E_{Fp} \]

\[ U = \frac{\sigma v_{th} N_t n_i^2 \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{n_i \left( \exp \left( \frac{2E_{Fn} - 2E_i - E_{Fp}}{2k_B T} \right) + \exp \left( \frac{E_{Fn} + E_{Fp} - 2E_{Fp}}{2k_B T} \right) + 2 \right)} \]
Simplifications

• Which leads to:

\[
U = \frac{\sigma v_{th} N_t n_i^2 \left( \exp \left( \frac{eV}{k_B T} \right) - 1 \right)}{n_i \left( \exp \left( \frac{E_{F_n} - E_{FP}}{2k_B T} \right) + \exp \left( \frac{E_{F_n} - E_{FP}}{2k_B T} \right) + 2 \right)}
\]

• The \( \exp(...) \) terms in the denominator are the same:

\[
\frac{\exp \left( \frac{eV}{k_B T} \right) - 1}{\exp \left( \frac{E_{F_n} - E_{FP}}{2k_B T} \right) + 1}
\]

\[
\frac{\exp \left( \frac{eV}{k_B T} \right) - 1}{\exp \left( \frac{E_{F_n} - E_{FP}}{2k_B T} \right) + 1}
\]

Recall our definition of Fermi-Level splitting:

\[ eV = E_{F_n} - E_{FP} \] (8)

Recall our definition of Fermi-Level splitting:

\[ U = \frac{\sigma v_{th} N_t n_i \left[ \exp \left( \frac{eV}{k_B T} \right) - 1 \right]}{2 \left( \exp \left( \frac{E_{F_n} - E_{FP}}{2k_B T} \right) + 1 \right)} \] (16)
Reverse Bias

i.e. $|V| \gg \frac{k_BT}{e}$

but $V$ is negative

$$U = \frac{\sigma v_{th} N_t n_i \left[ \exp \left( \frac{eV}{k_BT} \right) - 1 \right]}{2 \left[ \exp \left( \frac{eV}{2k_BT} \right) + 1 \right]}$$  \hspace{1cm} (16)

- What happens if $V \ll -k_BT/e$?
- Since $\lim_{x \to -\infty} [e^x] = 0$:

$$U \approx -\frac{1}{2} \sigma v_{th} N_t n_i = \frac{-n_i}{\tau_{SRH}}$$  \hspace{1cm} (17)

$$\tau_{SRH} = \frac{1}{2\sigma v_{th} N_t}$$  \hspace{1cm} (18)

- Notice that $U$ is now negative. This corresponds to carrier generation.

Forward Bias

$$U = \frac{\sigma v_{th} N_t n_i \left[ \exp \left( \frac{eV}{k_BT} \right) - 1 \right]}{2 \left[ \exp \left( \frac{eV}{2k_BT} \right) + 1 \right]}$$  \hspace{1cm} (16)

- What happens if $V \gg k_BT/e$?
- We know that $\lim_{x \to +\infty} [e^x] \gg 1$. i.e.:

$$\exp \left( \frac{eV}{k_BT} \right) - 1 \approx \exp \left( \frac{eV}{k_BT} \right) \exp \left( \frac{eV}{2k_BT} \right) + 1 \approx \exp \left( \frac{eV}{2k_BT} \right)$$

$$U = \frac{\sigma v_{th} N_t n_i \exp \left( \frac{eV}{k_BT} \right)}{2 \exp \left( \frac{eV}{2k_BT} \right)} \hspace{1cm} U = \frac{1}{2} \sigma v_{th} N_t n_i \exp \left( \frac{eV}{k_BT} \right) \exp \left( \frac{-eV}{2k_BT} \right)$$
Forward Bias

\[ U = \frac{1}{2} \sigma v_{th} N_t n_i \exp \left( \frac{eV}{k_B T} \right) \exp \left( \frac{-eV}{2k_B T} \right) \]

\[ U = \frac{1}{2} \sigma v_{th} N_t n_i \exp \left( \frac{eV}{k_B T} \right) \left[ 1 - \frac{1}{2} \right] \]

\[ U = \frac{1}{2} \sigma v_{th} N_t n_i \exp \left( \frac{eV}{2k_B T} \right) = \frac{n_i}{\tau_{SRH}} \exp \left( \frac{eV}{2k_B T} \right) \]  

(17)

• Notice that \( U \) is now positive. This corresponds to carrier recombination.

General Expression

• When describing recombination-generation rates generally we simply sum the two terms:

\[ U(V) = \frac{n_i}{\tau_{SRH}} \left[ \exp \left( \frac{eV}{2k_B T} \right) - 1 \right] \]  

(18)

• At zero bias \( U(V = 0) = 0 \).
• For reverse bias \( U(V < 0) < 0 \).
• For forward bias \( U(V > 0) > 0 \).
**RG-Current**

- To go from recombination-generation rate to recombination-generation (R-G) current density we integrate over the depletion width:

\[ J_{\text{gen}} = \int_0^W eUdx = qUW \]  

\[ J_{\text{gen}} = \frac{en_iW}{\tau_{SRH}} \left[ \exp \left( \frac{eV}{2k_BT} \right) - 1 \right] \]  

- Important aspects of this equation:
  - "Ideality factor" of 2 in the exponential
  - Lack of hard saturation in reverse bias due to the voltage dependence of the SRC width \( W \).

**Total Reverse Bias Current**

- The total reverse bias current consists of diffusion current (last Lecture) and generation current (this Lecture):

\[ J_R = J_{\text{diff}} + J_{\text{gen}} = \left[ \frac{eD_p p_n0}{L_p} + \frac{en_iW}{\tau_{SRH}} \right] \]  

- Use Law of Mass Action:

\[ p_n0 = \frac{n_i^2}{N_d} \]  

\[ J_R = \left[ \frac{eD_p n_i^2}{L_p N_d} + \frac{en_iW}{\tau_{SRH}} \right] \]
Total Reverse Bias Current

• Important points about the reverse bias current.

\[ J_R = \frac{eD_p n_i^2}{L p N_d} + \frac{en_i W}{\tau_{SRH}} \]  \hspace{1cm} (23)

• For semiconductors with large intrinsic carrier concentration (small \( E_g \)'s, e.g. Ge), \( J_{\text{diff}} \) dominates.

• For semiconductors with small \( n_i \) (wider \( E_g \)'s e.g. Si & GaAs), \( J_{\text{gen}} \) dominates.

• The reverse current does not saturate for \( J_{\text{gen}} \) current as it does for \( J_{\text{diff}} \) current.

• Deep level traps, usually near mid-gap, are required for SRH recombination-generation to be operative.

Ideality Factor

• We can generalize the ideal diode law to account for the R-G current as well as the diffusion current by introducing an ideality factor, \( n \)

\[ J = J_0 \exp \left( \frac{eV_d}{nk_B T} \right) - 1 \]  \hspace{1cm} (24)

• Usually \( 1 \leq n \leq 2 \).

• Situations where \( n > 2 \) do exist.

• They typically involve surface or interface generation-recombination, bulk defects, or the inapplicability of ‘standard’ semiconductor theory.
**Ideality Factor**

- For forward biases greater than a few $k_B T$ we can neglect the $-1$ term in the current-voltage equation:

$$J = J_0 \left[ \exp \left( \frac{eV_F}{nk_BT} \right) - 1 \right]$$

$$J \approx J_0 \exp \left( \frac{eV_F}{nk_BT} \right)$$

**High-Level Injection and Series Resistance**

- $I_0,GR$, $I_0,DIFF$ are forward bias
- $F = \text{forward bias}$
- $n = 1$ (DIFF), $n = 2$ (GR)
- $\tau_S$-dominated
- High-level injection
- Generation-recombination
- Diffusion current
- (we will come back to this)
High Level Injection

- At high current densities (forward biases or breakdown), several things start to happen. First consider band bending:

\[ E_C \]
\[ E_i \]
\[ E_V \]

\[ x \]

\[ V_F \]

\[ \Phi_{bi} - V_F \]

High Level Injection

- According to the ideal theory:
  - The forward bias decreases the band bending (i.e. the field across the SCR).
  - Thus, application of enough forward bias should eventually result in a flatband situation (no electric field across the SCR).
  - In reality this cannot happen.
    - Instead, part of the voltage drops across the quasi-neutral regions as well as the SCR.
    - The next slide illustrates this.
Series Resistance

• The simplest method of accounting for this voltage drop is to treat it as a series resistance, \( R_S \) and modify the ideal diode law as follows

\[
R_S = R_{S1} + R_{S2} \tag{25}
\]

\[
J = J_0 \left[ \exp \left( \frac{e[V_d - IR_S]}{n k_B T} \right) - 1 \right] \tag{26}
\]
Series Resistance

- At high current levels, we can no longer neglect voltage drop across the quasi-neutral regions:

\[ V_J = V - I R_S \]  \hspace{1cm} (27)

\[ J = J_0 \left[ \exp \left( \frac{eV_J}{n k_B T} \right) - 1 \right] \]  \hspace{1cm} (26)

High Level Injection

- Another implication of high-level injection is that the injected minority carrier density is no longer negligible compared to the majority carrier density.
- Actually, this is the definition of high-level injection
- Our low-level injection assumption is broken (Point 3, Slide 36, Lecture 4).
- For high level injection we say:

\[ p_n \approx n_n \]  \hspace{1cm} (28)

- The injected minority carrier density is approximately equal to the majority carrier concentration and can no longer be ignored.
Low Level Injection

- Recall what the carrier concentration looked like under forward bias and low-level injection:

  - Logarithmic
    - $p_n \approx N_A$
    - $n_p \approx n^2 N_A$
    - $n_{po} \approx N_A$
    - $p_{po} \approx n^2 N_A$

  - $\varepsilon = 0$
  - $\varepsilon \neq 0$
  - $\varepsilon = 0$

High Level Injection

- For high-level injection it will look more like this:

  - Logarithmic
    - $p_n \approx N_A$
    - $n_p \approx n^2 N_A$
    - $n_{po} \approx N_A$
    - $p_{po} \approx n^2 N_A$
High Level Injection

• Recall, from our definition of Fermi-Level splitting:

\[ p_n n_n = n_i^2 \exp \left( \frac{eV}{k_B T} \right) \]  

(9)

• For high-level injection we can say:

\[ p_n \approx n_n \]  

(28)

• Hence:

\[ p_n = n_i \exp \left( \frac{eV}{2k_B T} \right) \]  

(29)

• How does this affect the measured current density?

High Level Injection

• Recall from last time that current density is proportional to the first derivative of hole density with respect to position:

\[ J_p = eD_p \frac{dp_n}{dx} \]  

Equation (43) Lecture 4

• Hence we can say:

\[ J_p \propto \exp \left( \frac{eV}{2k_B T} \right) \]  

(30)

• I.e. we have an ideality factor of \( n = 2 \) for high level injection as well.
Threshold Voltage

- Consider the forward I-V characteristics for Ge, Si, and GaAs pn junctions:

\[ V_{th}^{(GaAs)} > V_{th}^{(Si)} > V_{th}^{(Ge)} \]

\[ E_g^{(GaAs)} > E_g^{(Si)} > E_g^{(Ge)} \]
Threshold Voltage

- Recall that $\phi_{bi}$ is the potential barrier for the injection of majority carriers under forward bias.

$$\phi_{bi} = \frac{k_B T}{e} \ln \left( \frac{n_{n0}p_{p0}}{n_i^2} \right)$$

- Clearly $\phi_{bi}$ increases with $E_g$ since $n_i$ decreases exponentially with $E_g$:

$$n_i = \sqrt{N_CN_V} \exp \left( \frac{-E_g}{2k_BT} \right)$$  \hspace{1cm} (31)

- $N_C$ Number density of conduction band states.
- $N_V$ Number density of valence band states.

Breakdown
Breakdown

- Though we have drawn the ideal I-V characteristics of the pn junction many times, we must now recognize that at sufficiently large reverse biases, the current increases abruptly.

- At high enough reverse bias, current increases.

\[ V = IR + V_{BD} \]

- \( V_{BD} \) is the breakdown voltage of the diode.

- I.e. we can choose \( R \) so:
  \[ IV_{BD} < P_{max} \]

I.e. the maximum voltage drop across the diode
Breakdown

• An example of where the reverse breakdown behavior is useful is in a Zener diode where the breakdown voltage may be used as a voltage reference for voltage regulator applications.

• There are three kinds of pn junction breakdown:
  • Thermal instability.
  • Tunneling (Zener).
  • Avalanche.

Thermal Instability

• Recall that for ideal diode theory, the reverse saturation current for a pn junction is:

\[ J_0 = \frac{eD_p n_i^2}{L_p N_D} + \frac{eD_n n_i^2}{L_n N_A} \]  

Equation (32)

• pn junctions that exhibit this type of reverse current behavior are fabricated using small bandgap, \( E_g \) materials with large intrinsic carrier concentration, \( n_i \).
Thermal Instability

- At large reverse bias ($V < 0$), a large amount of power ($P = -IV$) must be dissipated.
- This dissipated power will eventually increase the temperature $T$ of the semiconductor, which in turn will dramatically increase the current.
- This is because of the temperature dependence of intrinsic carrier concentration as we saw earlier:

$$n_i = \sqrt{N_C N_V} \exp\left(\frac{-E_g}{2k_B T}\right)$$ (31)

- Since $J_0 \propto n_i^2$.

- A larger current will cause more heating which will further increase the temperature, and so on...
- Thus, we have a positive feedback situation in which breakdown eventually occurs.
- Note that this mechanism is important for small $E_g$ semiconductors (e.g. Ge, InGaAs) at room temperature.
- It is not important at reduced temperatures.
Tunneling (Zener) Breakdown

- When both sides of a junction are heavily doped ($\gtrsim 10^{17}$ cm$^{-3}$) the depletion layer width is narrow.
- Upon application of a reverse bias there are a large density of filled VB states (on the p-side) at the same energy as a large density of empty CB states (n-side).
- These $e^-$ can lower their energy if they can transfer to the lower energy states. The transfer process is via quantum mechanical tunneling.

The likelihood of tunneling depends upon:
- The availability of empty and filled states at the same energy.
- The tunneling probability, which increases as the tunneling distance and barrier height decrease.

For Zener breakdown, $W$ must be $\leq 10$nm
Avalanche Breakdown

- Due to runaway impact ionization.
- \( e^- \) must build up enough energy to break bonds.
- Important when the doping concentration is not too large (so that Zener BD is not likely).

Small Reverse Bias

Large Reverse Bias

Ionization Rate

- Band to band impact ionization is usually characterized by the ionization rate: \( \alpha \).

\[
\alpha = \frac{e^- - h^+ \text{ created}}{\text{distance}} \tag{32}
\]

- Threshold energy for ionization in Si:
  - \( e^- \sim 3.6 \text{ eV} \)
  - \( h^+ \sim 5.0 \text{ eV} \)
Multiplication Factor

- Breakdown does not occur sharply at $V = -V_{BD}$.
- Another parameter which characterizes impact ionization / avalanche is the multiplication factor, $M$:

\[
M = \frac{1}{1 - \left[ \frac{V}{V_{BD}} \right]^m}
\]  

- $m$ is property of material.
- Avalanche breakdown is defined as occurring as $M \to \infty$.

Breakdown Voltages

- We will not derive them here, but Sze & Ng give a full derivation on p104-107 of the analytical expressions for the hole-initiated avalanche breakdown (BD) voltage.
- For a one-sided abrupt junction:

\[
V_{BD} = \frac{\epsilon_r \epsilon_0 \varepsilon_{crit}^2}{2eN_b}
\]  

- For a linearly-graded junctions:

\[
V_{BD} = \frac{4\epsilon_{crit}^{3/2}}{e \alpha} \sqrt{\frac{2\epsilon_r \epsilon_0}{ea}}
\]
Breakdown Voltages

- Breakdown is a completely reversible process – no harm to junction.
- Note that $V_{BD}$ increases as $E_g$ increases.
- This makes sense as carriers must gain more kinetic energy before they have energy to initiate impact ionization.
- For germanium and silicon $E_{BD} \approx 10^6$ V/cm.

Universal Expressions for $V_{BD}$

- These are derived empirically for the two specific types of junction we have studied.
- For an abrupt one-sided junction:
  $$V_{BD} \approx 60 \left( \frac{E_g}{1.1 \text{eV}} \right)^{3/2} \left( \frac{N}{10^{16} \text{cm}^{-3}} \right)^{-3/4}$$  \hspace{1cm} (36)
  
- For a linearly graded junction:
  $$V_{BD} \approx 60 \left( \frac{E_g}{1.1 \text{eV}} \right)^{6/5} \left( \frac{a}{3 \times 10^{20} \text{cm}^{-4}} \right)^{-2/5}$$  \hspace{1cm} (37)
Summary

- We looked at some of the ways in which the current-voltage behavior of pn-junctions can differ from the ideal diode equation we derived last time.

![Graph showing log(I_F) vs. V_F with different n values]

- We also spent a bit of time looking at what happens if we apply a very large reverse bias, and what causes breakdown.

Next Time...

- Time-Dependence of pn-Junctions.

![Diagrams showing the behavior of a p-n junction at different times]

- Reading: Section 2.5 of Sze & Ng (p114-117).