Lecture 9: Ideal Electrostatics, Capacitance, and Barrier Lowering

Sze And Ng: Chapter 3

Announcements

Literature Report 1

- Is online now.
- You will have 2 weeks to complete the report.
  - It is due Thursday February 11th at 8:30 am.
  - Email it to me at john.labram@oregonstate.edu.
- This report contributes 20% of the overall grade of the course.
- There are a total of 25 marks available.
- The report will consist of combination of short and long questions.
- Any questions, don’t hesitate to email me!
Last Time

- We looked at what happens when you bring a semiconductor into contact with a metal:

- We also dealt with issues relating to interface states:

Lecture 9

- Schottky Barrier Electrostatics.
- Schottky Depletion Capacitance.
- Barrier Lowering.
Electrostatics

- We want to be able quantify the band diagram.

**pn-Junction**

**Schottky Barrier**

- c.f. our pn junctions
Electrostatics

• The procedure we follow is similar to that we followed in Lecture 2 for pn junctions:

1. Determine the charge density: \( \rho(x) \).

2. Evaluate \( \mathcal{E}(x) \) using:

\[
\mathcal{E}(x) = \frac{1}{\varepsilon_r \varepsilon_0} \int_{-\infty}^{x} \rho(\tilde{x}) \, d\tilde{x} \quad (1)
\]

3. Evaluate \( \phi(x) \) using:

\[
\phi(x) = -\int_{-\infty}^{x} \mathcal{E}(\tilde{x}) \, d\tilde{x} \quad (2)
\]

4. Flip over \( \phi(x) \) to get energy band diagram.

2-Dimensional Charge Density

• We know the charge density of the right is due to donor density:

\[
\rho(0 \leq x \leq W_{dn}) = eN_D \quad (3)
\]

Where we have assumed that each donor provides a delocalized electron.

• The 2-dimensional charge density to the right of the interface is then:

\[
Q_{sc}(0 \leq x \leq W_{dn}) = eN_D W_{dn} \quad (4)
\]
Charge Balance

- From last time we know that charge is located in three parts of the junction:
  - Semiconductor depletion region, with density $Q_{SC}$.
  - Metal, with density $Q_M$.
  - Surface states, with density $Q_{SS}$.
- Recall, due to charge balance, we also stated that:
  \[ Q_{SC} = -(Q_M + Q_{SS}) \]  \[ (Q_M + Q_{SS}) = -eN_D W_{dn} \]  
- Hence we can say:

Charge Density

- We sketched out the charge density last time:
- We also showed that for typical doping densities:
  \[ \frac{W_{metal}}{W_{dn}} \approx 10^{-5} \]  
- For this reason we approximate the charge density in the metal and surface as a delta function:
  \[ \delta(x) = \begin{cases} 
  +\infty & x = 0 \\
  0 & x \neq 0 
\end{cases} \]  
  \[ \int_{-\infty}^{+\infty} \delta(x) dx = 1 \]
Electric Field

- Hence we describe the three dimensional charge density as:

\[
\rho(x) = \begin{cases} 
-eN_D W_{dn} \delta(x) & x = 0 \\
N_D & 0 \leq x \leq W_{dn} \\
0 & \text{elsewhere}
\end{cases}
\]  

(10)

- The electric field is:

\[
\mathcal{E}(x) = \frac{1}{\varepsilon_{rs} \varepsilon_0} \int_{-\infty}^{x} \rho(\xi) \, d\xi
\]

(1)

- Hence we can say:

\[
\mathcal{E}(x) = \frac{1}{\varepsilon_{rs} \varepsilon_0} \int_{-\infty}^{x} \left( eN_D - eN_D W_{dn} \delta(x) \right) \, d\xi
\]

- We handle \(\delta\)-functions slightly differently so we should separate the integral:

\[
\mathcal{E}(x) = \frac{1}{\varepsilon_{rs} \varepsilon_0} \int_{-\infty}^{x} eN_D \, d\xi - \frac{1}{\varepsilon_{rs} \varepsilon_0} \int_{-\infty}^{x} eN_D W_{dn} \delta(x) \, d\xi
\]

- \(\delta\)-functions are zero if and only if \(x = 0\), hence we can say:

\[
\frac{1}{\varepsilon_{rs} \varepsilon_0} \int_{-\infty}^{x} eN_D W_{dn} \delta(x) \, d\xi = \frac{eN_D W_{dn}}{\varepsilon_{rs} \varepsilon_0}
\]

- Aside: be careful with units when using \(\delta\)-functions (Strictly, they are distributions not functions!)
Electric Field

• Putting this back into our equation for $\mathcal{E}(x)$:

$$\mathcal{E}(x) = \frac{1}{\varepsilon_r \varepsilon_0} \int_{-\infty}^{x} eN_D \, d\bar{x} - \frac{eN_D W_{dn}}{\varepsilon_r \varepsilon_0}$$

• We know to the left of $x = 0$ there is no charge, therefore we change the lower limit:

$$\mathcal{E}(x) = \frac{1}{\varepsilon_r \varepsilon_0} \int_{0}^{x} eN_D \, d\bar{x} - \frac{eN_D W_{dn}}{\varepsilon_r \varepsilon_0}$$

• It is then easy to show:

$$\mathcal{E}(x) = \frac{eN_D}{\varepsilon_r \varepsilon_0} (x - W_{dn}) \quad \text{for } 0 \leq x \leq W_{dn} \quad (11)$$

Electric Field

• I.e. the electric field varies linearly across the depletion region.

• The maximum electric field is that at the interface:

$$\mathcal{E}_{max} = \mathcal{E}(x = 0)$$

$$\mathcal{E}_{max} = -\frac{eN_D W_{dn}}{\varepsilon_r \varepsilon_0} \quad (12)$$
Electrostatic Potential

- To get the electrostatic potential, we integrate the electric field over space:

\[ \phi(x) = - \int_{-\infty}^{x} \mathcal{E}(\bar{x}) \, d\bar{x} \quad (2) \]

- Again, we identify there is no field outside the depletion region:

\[ \phi(x) = - \int_{0}^{x} \mathcal{E}(\bar{x}) \, d\bar{x} \]

• Substitute from the electric field:

\[ \mathcal{E}(x) = \frac{eN_D}{\varepsilon_{rs}\varepsilon_0} (x - W_{dn}) \quad (11) \]

• Taking care with the signs:

\[ \phi(x) = \frac{eN_D}{\varepsilon_{rs}\varepsilon_0} \left( xW_{dn} - \frac{x^2}{2} \right) \quad \text{for } 0 \leq x \leq W_{dn} \quad (13) \]
Built in Potential

\[ \phi(x) = \frac{eN_D}{\epsilon_{rs}\epsilon_0} \left( xW_{dn} - \frac{x^2}{2} \right) \]  
(13)

for \( 0 \leq x \leq W_{dn} \)

- Recognize the built in potential is \( \phi_{bi} = \phi(W_{dn}) \)

\[ \phi_{bi} = \phi(W_{dn}) = \frac{eN_D}{\epsilon_{rs}\epsilon_0} \left( W_{dn}^2 - \frac{W_{dn}^2}{2} \right) \]

\[ \phi_{bi} = \frac{eN_D W_{dn}^2}{2\epsilon_{rs}\epsilon_0} \]  
(14)

Depletion Width

- Re-arranging (14) we get an expression for the width of the depletion region (which we call \( W_{dn} \rightarrow W \)).

\[ W = \frac{2\epsilon_{rs}\epsilon_0}{eN_D} \phi_{bi} \]  
(15)

- Notice this is also the equation for the depletion width a one-sided pn-junction!

- We can generalize this to account for an external applied voltage (\( V \)) by changing \( \phi_{bi} \rightarrow \phi_{bi} - V \):

\[ W = \frac{2\epsilon_{rs}\epsilon_0}{eN_D} (\phi_{bi} - V) \]  
(16)
**Built-In Potential**

- To evaluate this expression for the depletion region width of a Schottky barrier, you need to know the built-in voltage.
- Last time we found this is given by:
  \[ \phi_{bi} = \phi_{Bn} - \phi_n \]  
  \[ (17) \]
- Where:
  \[ \phi_n = (E_C - E_F) \bigg|_{\text{bulk}} \]  
  \[ (18) \]

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**Schottky Depletion Capacitance**
Capacitance

- Recall that capacitance-voltage (C-V) analysis is a useful method for testing a pn junction.
- C-V analysis consists of measuring the small-signal dynamic capacitance as a function of voltage.

Capacitance

- It is also useful for characterization of Schottky barriers.
- If you consider charge balance in a Schottky barrier, the dynamic depletion capacitance is just that of a voltage dependent parallel plate capacitor.
Capacitance

• We just evaluate the capacitance by describing the system as a parallel plate capacitor:

\[ C = \frac{\varepsilon_r \varepsilon_0}{W} \]  \hspace{1cm} (19)

• Substitute in Equation (16) for depletion width:

\[ C = \frac{\varepsilon_r \varepsilon_0 A}{2 \varepsilon_r s \varepsilon_0 (\phi_{bi} - V)} \]  \hspace{1cm} (20)

\[ C = \frac{C_0}{\sqrt{1 - \frac{V}{\phi_{bi}}} } \]  \hspace{1cm} (21)

• Where \( C_0 \) is the zero bias depletion capacitance:

\[ C_0 = \frac{\varepsilon_r \varepsilon_0 A}{\sqrt{2 \phi_{bi}} e N_D \varepsilon_r s \varepsilon_0} \]  \hspace{1cm} (22)

• Re-arranging (20) we can get:

\[ \frac{1}{C^2} = \frac{2}{e N_D \varepsilon_r s \varepsilon_0 A^2} (\phi_{bi} - V) \]  \hspace{1cm} (23)

• We can recognize this as the equation for a straight line.

• By plotting \( 1/C^2 \) against \( V \) we should hence get a straight line.
Capacitance

- We identify the gradient as:
  \[ m = -\frac{2}{eN_D\epsilon_r\epsilon_0 A^2} \]  
  \[ \text{(24)} \]
- And the intercept as
  \[ c = \phi_{bi} \]  
  \[ \text{(25)} \]

Note that there is no diffusion capacitance (or almost none), since a Schottky barrier is a majority carrier device.

This results in faster switching speeds and operating frequencies since you don’t have to wait for minority carrier profiles to build up or, especially, decay away.
Barrier Lowering

Method of Images

- We are going to talk about Barrier Lowering, also called Image Force Lowering (Section 3.2.4 Sze & Ng).
- We will exploit the Method of Images (or Method of Image Charges).
- A way of evaluating electric field strength due to charges above “perfect conductors”.
- We won’t go into details on the theory, but exploit its results. You can see more details here:
  - [https://www.youtube.com/watch?v=-AYHbjDf9sM](https://www.youtube.com/watch?v=-AYHbjDf9sM)
  - [https://www.youtube.com/watch?v=jippPv6Gz14](https://www.youtube.com/watch?v=jippPv6Gz14)
**Barrier Lowering**

- For an electron at the Fermi level of a metal to escape from the metal, it must acquire enough kinetic energy to overcome the work function.

  - However, some positive charge will be induced at the surface of the metal.
  - There cannot be any electric field inside of a (perfect) conductor.
    - Charges will always move to neutralize it.

**Method of Images**

- When an electron leaves the metal surface and is at a distance $x$ from the surface, we can find its potential energy by assuming that a positive image charge is located at a distance $-x$ into the metal.
Potential Energy

- Consider the work done ($E_{WD}$) in bringing this electron from a distance of $\infty$ from the surface to a distance $x$ from the surface.

$$E_{WD}(x) = \int_{\infty}^{x} F(\bar{x})d\bar{x} \quad (26)$$

- Where $F(x)$ is the force experienced by the charge at distance $x$ from the surface.

- We evaluate this as the Coulombic force acting between these two point charges:

$$F(x) = \frac{-e^2}{4\pi\varepsilon_0(2x)^2} = \frac{-e^2}{16\pi\varepsilon_0 x^2} \quad (27)$$

Potential Energy

- We can show that the work done ($E_{WD}$) in bringing this electron from a distance of $\infty$ from the surface to a distance $x$ from the surface is hence:

$$E_{WD}(x) = \frac{-e^2}{16\pi\varepsilon_0 x} \quad (28)$$

- Potential energy is (by definition) the work required to move a positive test charge from infinity to the point in question. Hence, the sign of the potential energy is negative.

$$PE_0(x) = \frac{-e^2}{16\pi\varepsilon_0 x} \quad (29)$$
Potential Energy

- Let’s plot this electron potential energy with no field.

\[ PE_0(x) = \frac{-e^2}{16\pi\epsilon_0 x} \]  

- So this image charge reduces the barrier vs a step function.

Potential Energy

- Also consider electric field:

\[ PE(x) = \frac{-e^2}{16\pi\epsilon_0 x} + e\mathcal{E}x \]  

- Where \( \mathcal{E} \) is the field due to \( \phi_{bi} \) or \( V \).
Potential Energy

• The textbook uses a similar notation in this case:

\[ E_x \]
\[ q_{\Delta \phi} \]
\[ q_{\phi_0} \]
\[ q_{\phi_m} \]

Vacuum

Metal

Image potential energy

Barrier Lowering

• This energy band diagram demonstrates that the barrier seen by an electron in the metal is lowered by the combination of:
  • The applied electric field.
  • The image force.
  • The barrier is lowered by an amount \( e\Delta \phi \).
  • The maximum barrier (minimum \( e\Delta \phi \)) occurs at \( x_m \).
  • We can show:

\[ x_m = \frac{e}{16\pi \epsilon_0 |\epsilon|} \quad (31) \]
\[ \Delta \phi = \frac{e|\epsilon|}{4\pi \epsilon_0} \quad (32) \]
Relative Permittivity

- These expressions for \(x_m\) and \(\Delta \phi\) were derived assuming the metal was adjacent to a vacuum (i.e. we describe the permittivity as \(\epsilon = \epsilon_0\)).
- For a metal-semiconductor interface, we replace \(\epsilon_0\) by \(\epsilon = \epsilon_0 \epsilon_r\):

\[
x_m = \sqrt{\frac{e}{16\pi \epsilon_0 \epsilon_r |\mathcal{E}|}} \quad (33)
\]
\[
\Delta \phi = \sqrt{\frac{e |\mathcal{E}|}{4\pi \epsilon_0 \epsilon_r}} \quad (34)
\]
- Normally when we talk about dielectric constant (relative permittivity) we mean then low-frequency \((f)\) value. But strictly speaking: \(\epsilon_r = \epsilon_r(f)\).

Aside: Dielectric Constant

- So what is \(\epsilon_r\)?
  - \(\epsilon_r\): the relative permittivity.
  - Also called \(\kappa\) is the dielectric constant.
- It takes account of the fact that the electric field (due to a certain voltage) is lower than it would be in a vacuum.
  - In a vacuum: \(\epsilon_r = \kappa = 1\).
  - For example \(\kappa = 3.9\) for SiO\(_2\).
  - I.e. the \(E\)-field in SiO\(_2\) due to an electrode should be \(\sim 25\%\) of what it would be in a vacuum.
Aside: Dielectric Constant

• Why is this the case?
• A material with $\kappa > 1$ will have some polarization properties.

By applying an external field, charges move, creating internal fields, and in-turn reducing the overall field.

This polarization cannot happen instantaneously.
• In general we must talk about $\kappa$ at a certain frequency.
Aside: Dielectric Constant

- Luckily a lot of semiconductors have profiles like the following:

- However, if the transit time from \( x = 0 \) to \( x = x_m \) is small compared to the dielectric relaxation time, you would expect that the semiconductor cannot fully polarize so that it would be more appropriate to use the high frequency dielectric constant.

Usually, use of the static dielectric constant is close enough, but you should be careful if you need a high degree of accuracy and if the semiconductor dielectric constant exhibits a large amount of frequency dispersion.
Barrier Lowering

- The barrier height seen by an electron in the metal increases with increasing forward bias and is decreases with increasing reverse bias.

\[ Sze & Ng \text{ Fig. 3.10, pg. 147} \]

Note that both the barrier height, \( \phi_{Bn}' \), and the built-in potential, \( V_{bi}' \), are reduced by the image force lowering:

\[ \phi_{Bn}' = \phi_{Bn} - \Delta \phi \quad \text{(35)} \]

\[ \phi_{bi}' = \phi_{Bn} - \Delta \phi - \phi_n \quad \text{(36)} \]

- Where:

\[ e\phi_n = (E_C - E_F) \bigg|_{\text{bulk}} \quad \text{(18)} \]
Summary

- Electrostatics, capacitance, and barrier lowering.

Next Time...

- How charges cross the interface, and current-voltage characteristics.