ROB 538: Multiagent Systems

Week 6, Lecture 1:
Computational Social Choice
(from “multiagent systems”, Weiss, ed.)

Reading:
Chapters 6

Announcements:
Midterm Exam: 11/9
HW 2 due today (bonus: have 50 agents for problem 3.b)

Motivation

• What is social choice?
  - How to aggregate possibly conflicting preferences into collective choices in a fair and satisfactory way?

• voting (e.g., political, but also wikipedia, facebook, debian)
• resource allocation, fair division (e.g., cake cutting)
• coalition formation, matching (e.g., house allocation, college admission)
• webpage ranking (e.g., search engine aggregators, pagerank algorithm)
• collaborative filtering (e.g., amazon or ebay)
Motivation

- Key components of social choice
  - Autonomous agents (e.g., human or software agents)
  - A set of alternatives (usually finitely many)
  - Preferences over alternatives
  - Aggregation functions

Key Questions

- What does it mean to make rational choices?
- Which formal properties should an aggregation function satisfy?
- Which of these properties can be satisfied simultaneously?
- How difficult is it to compute collective choices?
- Can voters benefit by lying about their preferences?
Voting

• Assuming you know who you like best, who should you vote for?

• Should you try to make a strategic vote?

Plurality

• The candidate with the most votes wins

• Simple, most widely-used approach

• Is it the best?
  – What does that even mean?
Plurality

- Consider a *preference profile* with 21 voters, who rank four alternatives as in the table below.

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>a</td>
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<td>a</td>
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</tbody>
</table>

- Alternative *a* is the unique plurality winner despite the fact that
- a majority of voters think *a* is the worst alternative,
- *a* loses against *b, c, and d* in pairwise majority comparisons, and
- if the preferences of all voters are reversed, *a* still wins.

Five common voting rules

- Plurality
  - Alternative ranked first by most voters
- Borda
  - The most preferred alternative of each voter gets $k-1$ points, the second most preferred $k-2$, etc. the one with highest accumulated score wins
- Plurality with runoff
  - Two alternatives ranked first by most, face of in a majority runoff
- Instant runoff
  - Alternative ranked first by most voters eliminated. Repeat until one candidate remains
- Sequential majority comparison (SMC)
  - Alternative that wins a sequence of pairwise comparisons
Borda Count

- Weighted voting: The most preferred alternative of each voter gets $k-1$ points, the second most preferred $k-2$, etc. the one with highest accumulated score wins
  - 4 pts for top choice
  - 4 pts for second choice
  - 2 pts
  - 1 pt
  - 0 pts for least favorite choice

- Slovenia uses this type of voting
- Car racing (formula 1) determines winner this way
- Skiing determines winner this way

Plurality with Runoff

- One round may not capture preferences
- Top two vote getters in round face off in round 2
- Winner of round 2 is elected
  - France uses this type of voting for presidential election
Instant Runoff

• Why only two rounds?
• Why hold multiple elections?
• Ask for vote preferences
  – Delete least vote getter, reassign their votes to send choice
  – Keep going till one candidate gets majority

  – Australia, Ireland use this type of voting
  – Academy awards

Sequential Majority Comparison

• Present two alternatives
• Majority wins
• Repeat
  – US congress uses this process for bill adoption.
    • Bill amendments are voted on
    • Bill is voted on
  – Order?
### Sequential Majority Comparison

<table>
<thead>
<tr>
<th></th>
<th>203</th>
<th>116</th>
<th>116</th>
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<tbody>
<tr>
<td>1st choice</td>
<td>No Bill</td>
<td>Amended Bill</td>
<td>Original Bill</td>
</tr>
<tr>
<td>2nd choice</td>
<td>Amended Bill</td>
<td>Original Bill</td>
<td>No Bill</td>
</tr>
<tr>
<td>3rd choice</td>
<td>Original Bill</td>
<td>No Bill</td>
<td>Amended Bill</td>
</tr>
</tbody>
</table>

- **Vote 1:** Amend bill or not  
  Bill is amended 319 to 116
- **Vote 2:** Pass bill or not  
  Bill is defeated 319 to 116

Outcome: No Bill

#### Actual Outcome

### Example Preference Profile

```
<table>
<thead>
<tr>
<th>33%</th>
<th>16%</th>
<th>3%</th>
<th>8%</th>
<th>18%</th>
<th>22%</th>
</tr>
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<tr>
<td>a</td>
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<td>a</td>
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</tr>
</tbody>
</table>
```
Who is the winner?

- Plurality?
- Borda?
- Sequential Majority?
- Instant-runoff?
- Plurality with runoff?

Plurality

<table>
<thead>
<tr>
<th>33%</th>
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<tbody>
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<td>a</td>
<td>b</td>
<td>c</td>
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</tbody>
</table>

- a wins 33%
Plurality with Runoff

- a vs. e
- e wins 64% to 36%

### Instant runoff

<table>
<thead>
<tr>
<th>33%</th>
<th>16%</th>
<th>3%</th>
<th>8%</th>
<th>18%</th>
<th>22%</th>
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</table>

- Round 2: a:33% b:16% d:21% e:30%
- Round 3: a:33% d:37% e:30%
- Round 4: a:33% d:67%
Borda Count

<table>
<thead>
<tr>
<th></th>
<th>33%</th>
<th>16%</th>
<th>3%</th>
<th>8%</th>
<th>18%</th>
<th>22%</th>
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<tbody>
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</tbody>
</table>

- a: $4 \times 33 + 1 \times 3 = 135$
- b: $4 \times 16 + 3 \times 33 + 2 \times (3 + 8 + 22) + 1 \times 18 = 247$
- c: $4 \times (3 + 8) + 3 \times 22 + 2 \times (33 + 16 + 18) = 244$
- d: $4 \times 18 + 3 \times (16 + 3) + 1 \times (33 + 8 + 22) = 192$
- e: $4 \times 22 + 3 \times (8 + 18) + 1 \times (16) = 182$

Pairwise Comparison

- a vs. c: c (67%)
- b vs. c: c (51%)
- d vs. c: c (66%)
- e vs. c: c (60%)
Who is the winner?

- Plurality: a wins
- Borda: b wins
- Sequential Majority: c wins
- Instant-runoff: d wins
- Plurality with runoff: e wins

Desirable Properties

- Anonymity
  - The voting rule treats voters equally
- Neutrality
  - The voting rule treats candidates equally
- Monotonicity
  - A chosen alternative will still be chosen if it rises in individual preference rankings (all else being unchanged)
- Pareto optimality
  - An alternative (A) will not be chosen if there exists another alternative (B) such that all voters prefer alternative B to alternative A.
Desirable Properties

<table>
<thead>
<tr>
<th>Anonymity</th>
<th>Neutrality</th>
<th>Monotonicity</th>
<th>Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Borda</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Plurality w/ runoff</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Instant-runoff</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>SMC</td>
<td>✓</td>
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</tbody>
</table>

Monotonicity of Runoffs

- Top 2 are a (6) and b (6)
- Runoff:
  - a wins 11 to 6
Monotonicity of Runoffs

- a was winner. Now 2 more people ranked a as their favorite
- Top 2 are a (8) and c (5)
- Runoff:
  - a loses to c 9 to 8!
  - a lost to c because people switched their votes from b to a!

Strategic Manipulation

- So far, we assumed that the true preferences of all voters are known.
- This is an unrealistic assumption because voters may be better off by misrepresenting their preferences.

- Plurality winner a
  - b wins if the last two voters vote for b, whom they prefer to a.

- How about Borda?
  - a’s score: 9, b’s score: 14, c’s score: 13, d’s score: 6
  - c wins if the voters in the second column, who prefer c to b, move b to the bottom.
Gibbard-Satterthwaite Theorem

• Why is manipulation undesirable?
  - Spending energy and resources on manipulative activities will be rewarded.
  - Manipulative skills are not spread evenly across the population.
  - Predictions or theoretical statements about voting rules become extremely difficult.

• Every reasonable voting rule is prone to manipulation whenever there are more than two alternatives.
  - Gibbard-Satterthwaite impossibility theorem (1973/75)

• Research in computational social choice has investigated the question of whether manipulation can be made computationally difficult.

Gibbard-Satterthwaite Theorem

• In a vote with 3 or more candidates, one of three things must be true:
  - Vote is dictatorial
  - There is a candidate who cannot win under any circumstance
  - Voting is susceptible to tactical voting

Since 1 and 2 are not compatible with real life voting, we are left with:

Voting is susceptible to tactical voting
Hardness of Manipulation

• Finding a beneficial manipulation for the following voting rules is NP-hard:
  - Second-order Copeland (Bartholdi, Tovey, and Trick; 1989)
  - Instant-runoff (Bartholdi and Orlin; 1991)
  - Nanson’s rule (Narodyska et al.; 2011)

• Many more similar results for weighted voting and coalitional manipulation.
  - Key problem: NP-hardness is a worst-case measure
  - A string of recent results has cast doubt on this strand of research, culminating in work by Isksson et al. (2010).
  - Essentially, they show that for every efficiently computable, neutral voting rule, a manipulable preference profile with a corresponding manipulation can easily be found.

Probabilistic Voting Rules

• Another idea to circumvent the Gibbard-Satterthwaite impossibility is to introduce randomization.

• Probabilistic voting rules yield probability distributions (so-called lotteries) over alternatives.
  - Random dictatorship: Pick a voter a random (independently of the voters’ preferences) and choose her favorite alternative.

• Unfortunately, there is another far-reaching negative result.

• Whenever there are more than two alternatives, every non-manipulable, Pareto-optimal, probabilistic voting rule has to be a random dictatorship (Gibbard; 1977).
Strategic Abstaining

• Consider the following preference profile and plurality with runoff.
  - Alternative a wins.
  - If two voters of the last column do not vote, c wins.
  - These voters prefer c to a.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>c</th>
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<tbody>
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<td>c</td>
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<td>3</td>
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• Voters in the last column are better off by abstaining i.e., by not voting at all.

• Plurality and Borda are resistant to strategic abstention.

• Most other voting rules suffer from strategic abstention.

Other Voting Rules

• Condorcet winner:
  - If an alternative wins against every other alternative in pairwise majority comparisons, it is called a Condorcet winner.

• Young’s rule:
  - yields alternatives that can be made a Condorcet winner by removing as few voters as possible.
  - Computing Young winners is NP-hard!

• Approval voting
  - Rather than having complete preference rankings, voters only approve or disapprove of alternatives.
  - The alternative with the most approvals win.

• Range voting
  - Voters assign up to 100 points to each alternative.
  - Alternatives with maximal scores win.
Fair vs. Efficient

- End of chapter 6

Questions