HWS

$\overline{x} = 20.82$

$sd = 4.15$

$Min = 7$

$Max = 25$

Chapter 14.
Independent Demand Inventory Systems

Inventory:
Inventory may be considered an accumulation of a commodity that will be used to satisfy some future demand for that commodity.

Management of inventory can be classified into two groups.
1. Independent demand inventory management

Demand for these items are primarily influenced by factors that are independent of firm's decisions.

- Finished products in mfg & services
- Spare parts in maintenance.

2. Dependent demand inventory management

Demand for these items are directly influenced by internal factors and are well within the control of the firm company.

- Also called mfg. inventory.
- E.g.: Raw materials & component items that go into the finished product.

Types of inventory

1. Finished product:
   - Order is placed now. It is going to take some time to produce and supply the product.
   - Back ordering not permitted.

Demand: 10, 25, 15, 40 (next 4 weeks)

- Maintaining production at an avg. level may be advantageous; less costly.
- A customer is in a position to see the end product, if inventories are held.
2) In-progress inventories.
   - Enables uncoupling processing steps/stages.
   - Increases flexibility.

   + Processing rates of steps do not necessarily have to be equal.
   + A reduction in material handling and production cost may be realized by handling WIP in large batches.

3) Raw materials inventory.
   - Supplies may not be in a position to supply the raw materials exactly at the time they are needed.
   - Quantity discount/price needed.

   \[ Q, < 500 \quad \text{unit price} = \$1/\text{unit} \]
   \[ 500 \leq Q, < 1000 \quad \text{unit price} = \$0.9/\text{unit} \]
   \[ Q, \geq 1000 \quad \text{unit price} = \$0.8/\text{unit} \]

   \( \Rightarrow \) may be advantageous to choose the 3rd range even if the req is less.

   \( \Rightarrow \) Receiving large shipments might be beneficial (reduced material handling & transportation costs).
Two fundamental questions in inventory mgmt.

1) How much to order?
2) When to place an order?

The behaviour of independent demand inventory can be demonstrated by two models:

1) Fixed order quantity system: order quantity is held constant.
2) Fixed order period system: time between orders is constant.

Economic Order Quantity Model / EOQ Model (Model I)

\[
D = \text{annual demand for product/item} \quad \text{(units/yr)}
\]
\[
Q = \text{order quantity} \quad \text{(units per order)}
\]

2) a decision variable
\[
C = \text{cost of carrying a unit of item in inventory for one year} \quad \text{($/unit/yr$)}
\]
\[
S = \text{ordering cost or setup cost} \quad \text{($/order$)}
\]
\[
TSC = \text{Total annual stocking costs} \quad \text{($/yr$)}
\]
\[
TSC = \text{Annual carrying cost} + \text{Annual ordering cost}
\]
Assumptions:
* no safety stock,
* instantaneous replenishment,
* usage rate is constant,
* lead time is constant,

\[ \text{Avg. inventory level} = \frac{\text{Max. inventory} + \text{Min. inventory}}{2} \]

\[ = \frac{Q + 0}{2} = \frac{Q}{2} \]

(4.1) Annual carrying cost = \( \frac{\text{Avg. inv. level}}{2} \times \text{Carrying cost} \)

\[ = \frac{Q}{2} \times C \]

(4.2) Annual ordering cost = \( \text{Order per year} \times \text{Ordering cost} \)

\[ = \left( \frac{D}{Q} \right) \times S \]

\[ TSC = X_1 + X_2 \]
\[ = \frac{Q \times C}{2} + \frac{D}{Q} \times S \]
2. a) 
\[ D = 2,500,000 \text{ tons} \]
\[ C = 0.35(122.5) = \$42.875 \text{ per ton per year} \]
\[ S = \$1595 \text{ per order} \]
\[ EOQ = \sqrt{\frac{2DS}{C}} = \sqrt{\frac{2(2,500,000)(1595)}{42.875}} \]
\[ = 13,638.395 \text{ tons per order.} \]

b) \[ TSC = \left(\frac{Q}{2}\right)C + \left(\frac{D}{Q}\right)S \]
\[ = \left(13,638.395/2\right)(42.875) + \left(\frac{2,500,000}{13,638.395}\right)(1595) \]
\[ = 292,373.09 + 292,373.11 \]
\[ = \$584,746.20 \]

c) \[ \text{Orders per year} = \frac{D}{Q} \]
\[ = \frac{2,500,000}{13,638.395} \]
\[ = 183.3 \text{ orders per year.} \]
d). Time between orders.

\[ \text{(Days per year)} / \text{(Orders per year)} \]

\[ = (365) / (\frac{D}{Q}) = \frac{365}{183.3} \]

\[ = 1.99 \text{ or about 2 days.} \]

Model II - EOQ with gradual deliveries or EOQ with finite supply.

Additional notations:

- \( d \) = demand rate at which units are used out from inventory (units/time period)
- \( P \) = supply rate at which units are either produced or supplied to inventory (units/time period).

Model I

\[ \tan 90^\circ = \infty \]
\[ \text{Supply rate} = \infty \]

Model II

in-house mfg.

feeder dept.

\[ \text{rate of production} = 10 \text{ cc/min} \]
\[ \text{rate of consumption} = 15 \text{ cc/min} \]
Model II

\[ \text{Slope} = p - d \quad (q > d) \]

Let

\[ \text{A} = \text{supply/consumption begin} \]

\[ \text{B} = \text{supply ends/consumption continues} \]

\[ AD = \text{busy time} = \frac{Q}{p} \]

\[ DC = \text{idle time} = \left( \frac{x}{d} \right) = \frac{Q}{pd} (p-d) \quad \text{slope} = \frac{x}{AD} \]

Note:

\[ \frac{x}{Q} = (p-d) \]

\[ \frac{Q}{p} \]

\[ x = \left( \frac{Q}{p} \right) (p-d) \]

Avg. inventory = \[ \frac{x + D}{2} = \frac{1}{2} x \]

= \[ \frac{Q}{2p} (p-d) \]
Annual carrying cost = \( \text{Avg. inv. level} \times \text{carrying cost} \)

\[
= \frac{Q}{2p} (p - d) \times C
\]

Annual ordering cost = \( \text{Orders per year} \times \text{Ordering cost} \)

\[
= \left( \frac{D}{Q} \right) \times S.
\]

\[
TSC = Q \left( \frac{2(p - d)}{p} \right) \times C + \frac{D}{Q} \times S.
\]

\[
dQ = \frac{2(p - d)}{2p} \times C - \frac{DS}{Q^2}.
\]

\[
Q^2 = \frac{2DS}{C} \left( \frac{p}{p - d} \right)
\]

\[
Q = \sqrt{\frac{2DS}{C} \left( \frac{p}{p - d} \right)}
\]

\[
\sigma \sigma Q = \sqrt{\frac{2DS}{C} \left( \frac{p}{p - d} \right)}.
\]
Here

\[ D = 500,000 \text{ barrels/yr} \]
\[ S = \$7,500 \text{ order} \]
\[ C = \$0.25 \text{ (ac)/barrel/yr} \]
\[ P = 10,000 \text{ barrels/day} \]
\[ d = 5,000 \text{ barrels/day} \]
\[ ac = \$22.5 \text{ per barrel} \]

(a) \[ \text{EOQ} = \sqrt{\frac{2DS}{C}} \left( \frac{P}{P-d} \right) \]
\[ = \sqrt{\frac{2(500,000)(7500)}{0.25(22.5)}} \left( \frac{10000}{10000-5000} \right) \]
\[ = 51,639.778 \text{ barrels/yr} \]

(b) \[ \text{TSC} = \frac{Q}{2} \left[ \frac{PD}{P} \right] + \left( \frac{D}{Q} \right) S \]
\[ = \frac{51,639.778}{2} \left[ \frac{5000}{5000} \right] + \left( \frac{10000-5000}{10000} \right) \times 0.25 \times 22.5 \]
\[ = 72,618.44 + 72,618.44 \]
\[ = 145,236.88 \text{ per year} \]