CS 161
Intro to CS I

Recursion
Recursion

• What is it?
  – Function that calls itself 1 or more times (directly or indirectly)
  – Has 1 or more base case for stopping
  – Inductive reasoning: general case must eventually be reduced to a base case
Example: Drawing Rectangles

• Iterative Solution:

```cpp
void draw_rect(int i) {
    for( ; i > 0; i--){
        cout << "*****" << endl;
        cout << "*    *" << endl;
        cout << "*    *" << endl;
        cout << "*****" << endl << endl;
    }
}
```
Example: Drawing Rectangles

- Recursive Solution

```cpp
void draw_rect(int i) {
    if(i>0) { // Base case
        draw_rect(--i); // Recursive call
        cout << "*****" << endl;
        cout << "*     *" << endl;
        cout << "*****" << endl;
    }
}
```
What is different when we call after?

• Recursive Solution
void draw_rect(int i) {
    if(i>0) {  //Base case
        cout << "******" << endl;
        cout << "*         *" << endl;
        cout << "******" << endl << endl;
        draw_rect(--i);  //Recursive call
    }
}

Example: Factorial

- Definition

\[ 0! = 1; \]
\[ n! = n \times (n-1) \times \ldots \times (n-(n-1)) \times 1 = n \times (n-1)! ; n > 0 \]
Iterative Factorial

factorial(0) = 1;
factorial(n) = n*n-1*n-2*…*n-(n-1)*1;

long factorial(int n) {
    long fact;
    if(n==0)
        fact=1;
    else
        for(fact=n; n > 1; n--)
            fact=fact*(n-1);
    return fact;
}
Recursive Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

long factorial(int n) {
    if (n == 0)     // Base case
        return 1;
    else
        return n * factorial(n - 1);    // Recursive call
}
Computing Factorial Iteratively

\[ \text{factorial}(4) \]

\[
\text{factorial}(o) = 1; \\
\text{factorial}(n) = n \times (n-1) \times \ldots \times 2 \times 1;
\]
Computing Factorial Iteratively

factorial(4) = 4 * 3

factorial(0) = 1;
factorial(n) = n*(n-1)*...*2*1;
Computing Factorial Iteratively

\[
\text{factorial}(4) = 4 \times 3 \\
= 12 \times 2
\]

\[
\text{factorial}(0) = 1; \\
\text{factorial}(n) = n \times (n-1) \times \ldots \times 2 \times 1;
\]
Computing Factorial Iteratively

\[
\text{factorial}(4) = 4 \times 3 \\
= 12 \times 2 \\
= 24 \times 1
\]

\[
\text{factorial}(0) = 1; \\
\text{factorial}(n) = n \times (n-1) \times \ldots \times 2 \times 1;
\]
Computing Factorial Iteratively

\[
\text{factorial}(4) = 4 \times 3 \\
= 12 \times 2 \\
= 24 \times 1 \\
= 24
\]

factorial(0) = 1;

\[
\text{factorial}(n) = n \times (n-1) \times \ldots \times 2 \times 1;
\]
Computing Factorial Recursively

factorial(4)

factorial(0) = 1;
factorial(n) = n * factorial(n-1);
Computing Factorial Recursively

\[
\text{factorial}(4) = 4 \times \text{factorial}(3)
\]

\[
\text{factorial}(0) = 1;
\]

\[
\text{factorial}(n) = n \times \text{factorial}(n-1);
\]
Computing Factorial Recursively

factorial(4) = 4 * factorial(3)
    = 4 * ( 3 * factorial(2))
Computing Factorial Recursively

factorial(4) = 4 * factorial(3)
  = 4 * (3 * factorial(2))
  = 4 * (3 * (2 * factorial(1)))

factorial(0) = 1;
factorial(n) = n*factorial(n-1);
Computing Factorial Recursively

\[
\text{factorial}(4) = 4 \times \text{factorial}(3) \\
= 4 \times (3 \times \text{factorial}(2)) \\
= 4 \times (3 \times (2 \times \text{factorial}(1))) \\
= 4 \times (3 \times (2 \times (1 \times \text{factorial}(0))))
\]
Computing Factorial Recursively

\[ \text{factorial}(4) = 4 \times \text{factorial}(3) \]
\[ = 4 \times (3 \times \text{factorial}(2)) \]
\[ = 4 \times (3 \times (2 \times \text{factorial}(1))) \]
\[ = 4 \times (3 \times (2 \times (1 \times \text{factorial}(0)))) \]
\[ = 4 \times (3 \times (2 \times (1 \times 1))) \]
Computing Factorial Recursively

\[ \text{factorial}(4) = 4 \times \text{factorial}(3) \]
\[ = 4 \times (3 \times \text{factorial}(2)) \]
\[ = 4 \times (3 \times (2 \times \text{factorial}(1))) \]
\[ = 4 \times (3 \times (2 \times (1 \times \text{factorial}(0)))) \]
\[ = 4 \times (3 \times (2 \times (1 \times 1))) \]
\[ = 4 \times (3 \times (2 \times 1)) \]
Computing Factorial Recursively

\[
\text{factorial}(4) = 4 \times \text{factorial}(3) \\
= 4 \times (3 \times \text{factorial}(2)) \\
= 4 \times (3 \times (2 \times \text{factorial}(1))) \\
= 4 \times (3 \times (2 \times (1 \times \text{factorial}(0)))) \\
= 4 \times (3 \times (2 \times (1 \times 1))) \\
= 4 \times (3 \times (2 \times 1)) \\
= 4 \times (3 \times 2)
\]

\[
\text{factorial}(0) = 1; \\
\text{factorial}(n) = n \times \text{factorial}(n-1);
\]
Computing Factorial Recursively

factorial(4) = 4 * factorial(3)
  = 4 * (3 * factorial(2))
  = 4 * (3 * (2 * factorial(1)))
  = 4 * (3 * (2 * (1 * factorial(0))))
  = 4 * (3 * (2 * (1 * 1)))
  = 4 * (3 * (2 * 1))
  = 4 * (3 * 2)
  = 4 * 6

factorial(0) = 1;
factorial(n) = n*factorial(n-1);
Computing Factorial Recursively

\[\text{factorial}(4) = 4 \times \text{factorial}(3)\]
\[= 4 \times (3 \times \text{factorial}(2))\]
\[= 4 \times (3 \times (2 \times \text{factorial}(1)))\]
\[= 4 \times (3 \times (2 \times (1 \times \text{factorial}(0))))\]
\[= 4 \times (3 \times (2 \times (1 \times 1)))\]
\[= 4 \times (3 \times (2 \times 1))\]
\[= 4 \times (3 \times 2)\]
\[= 4 \times 6\]
\[= 24\]
Differences

• Pros
  – Readability

• Cons
  – Efficiency
  – Memory
Recursive Factorial

factorial(4) → Executes factorial(4)

Step 0: executes factorial(4)
Step 1: executes factorial(3)
Step 2: executes factorial(2)
Step 3: executes factorial(1)
Step 4: executes factorial(0)
Step 5: return 1
Step 6: return 1
Step 7: return 2
Step 8: return 6
Step 9: return 24
Recursive Factorial

return 1
factorial(4)
return 4 * factorial(3)
return 3 * factorial(2)
return 2 * factorial(1)
return 1 * factorial(0)

Step 0: executes factorial(4)
Step 1: executes factorial(3)
Step 2: executes factorial(2)
Step 3: executes factorial(1)
Step 4: executes factorial(0)
Step 5: return 1
Step 6: return 1
Step 7: return 2
Step 8: return 6

Executes factorial(3)
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

Step 3: executes factorial(1)

Step 4: executes factorial(0)

Step 5: return 1

Step 6: return 1

Step 7: return 2

Step 8: return 6

Executes factorial(2)
Recursive Factorial

factorial(4)

return 4 \* factorial(3)

Step 1: executes factorial(3)

return 3 \* factorial(2)

Step 2: executes factorial(2)

return 2 \* factorial(1)

Step 3: executes factorial(1)

Step 4: executes factorial(0)

Step 5: return 1

Step 6: return 1

Step 7: return 2

Step 8: return 6

Step 9: return 24

Executes factorial(1)

Stack

Space Required for factorial(2)

Space Required for factorial(3)

Space Required for factorial(4)

Main method
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

return 2 * factorial(1)

Step 3: executes factorial(1)

return 1 * factorial(0)

Step 5: return 1

Step 6: return 1

Step 7: return 2

Step 8: return 6

Stack

Executes factorial(0)

Space Required for factorial(1)

Space Required for factorial(2)

Space Required for factorial(3)

Space Required for factorial(4)

Main method
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)
return 4 * factorial(3)

Step 1: executes factorial(3)
return 3 * factorial(2)

Step 2: executes factorial(2)
return 2 * factorial(1)

Step 3: executes factorial(1)
return 1 * factorial(0)

Step 4: executes factorial(0)
return 1

returns 1

Space Required

Stack

for factorial(0)
for factorial(1)
for factorial(2)
for factorial(3)
for factorial(4)
Main method
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

return 2 * factorial(1)

Step 3: executes factorial(1)

return 1 * factorial(0)

Step 5: return 1

Step 4: executes factorial(0)

returns factorial(0)

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Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

return 2 * factorial(1)

Step 3: executes factorial(1)

return 1 * factorial(0)

Step 4: executes factorial(0)

Step 5: return 1

Step 6: return 1

Step 7: return 2

Step 8: return 6

returns factorial(1)
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

return 2 * factorial(1)

Step 3: executes factorial(1)

return 1 * factorial(0)

Step 4: executes factorial(0)

return 1

Step 5: return 1

Stack

OSU Oregon State University
Recursive Factorial

factorial(4)

Step 0: executes factorial(4)

return 4 * factorial(3)

Step 1: executes factorial(3)

return 3 * factorial(2)

Step 2: executes factorial(2)

return 2 * factorial(1)

Step 3: executes factorial(1)

return 1 * factorial(0)

Step 4: executes factorial(0)

returns factorial(3)

Step 5: return 1

Step 6: return 2

Step 7: return 3

Step 8: return 6

Main method
Recursive Factorial

Step 0: executes factorial(4)

Step 1: executes factorial(3)

Step 2: executes factorial(2)

Step 3: executes factorial(1)

Step 4: executes factorial(0)

Step 5: return 1

Step 6: return 1

Step 7: return 2

Step 8: return 6

Step 9: return 24

returns factorial(4)