

FINAL EXAMINATION
ECE 580

Dec.10, 2014
Open Book

1. a. Find the cascade ($ABCD$) matrix of the two-port shown in Fig. 1a.
- b. Find the cascade matrix for the circuit of Fig. 1b.

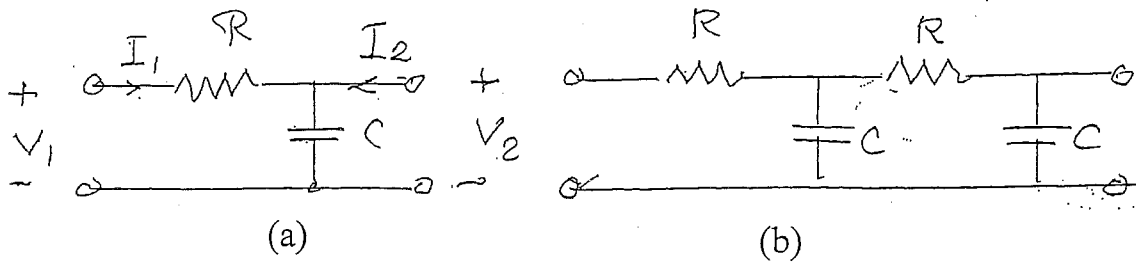


Fig. 1

2. At port 1 of a network, R_1 , Z_1 and S_{11} are known (Fig. 2). The network is redesigned such that R_1 remains the same, but S_{11} changes sign. What is the relation between the original Z_1 and the new one Z_1' ?

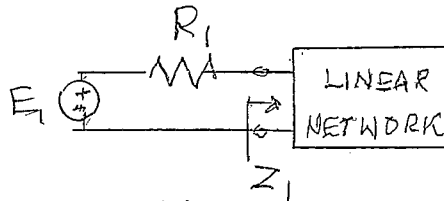


Fig. 2

3. a. Find the two transfer functions $H_i(s) = V_{out,i}/V_{in}$, $i = 1, 2$ of the two-port shown in Fig. 3.
- b. What are the pole frequency ω_p and the pole Q ?
- c. Assume $C_1/G_{m1} = 4C_2/G_{m2}$. What are the natural frequencies (poles)?
- d. Under the same assumption, where is the 3 dB frequency of $|H_2(j\omega)|$?

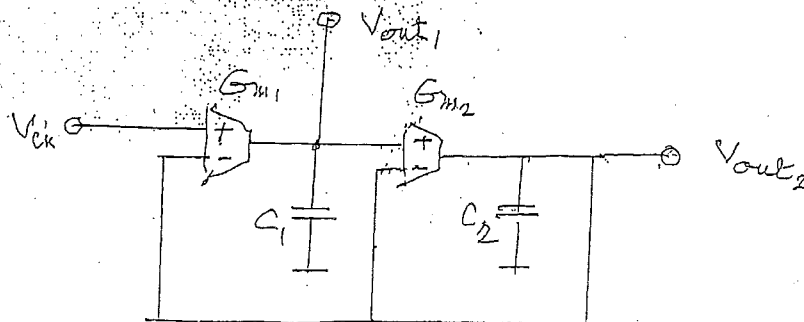


Fig. 3

Solutions

①

1 a. $I_1 = sC V_2 + I_2$

$$V_1 = R I_1 + V_2 = R (sC V_2 + I_2) + V_2$$

$$\tilde{\mathbf{I}} = \begin{bmatrix} sRC + 1 & +R \\ sC & +1 \end{bmatrix}$$

b. Simplest: $\tilde{\mathbf{T}}^2 = \begin{bmatrix} (sRC + 1)^2 + sRC R(sRC + 1) + R^2 & \\ sC(sRC + 1) + sC & sRC + 1 \end{bmatrix}$

2. $\frac{z_1 - R_1}{z_1 + R_1} = -\frac{z_1' - R_1}{z_1' + R_1} \Rightarrow z_1^2 = R_1^2 / z_1'$

3. Denote $T_i \triangleq C_i / G_{mi}$, $i = 1, 2$,

$$V_{out1} = \frac{G_{m1}}{sC_1} [V_{in} - V_{out2}] = \frac{1}{sT_1} (V_{in} - V_{out2})$$

$$V_{out2} = \frac{G_{m2}}{sC_2} (V_{out1} - V_{out2}) = \frac{1}{sT_2} \left[\frac{1}{sT_1} (V_{in} - V_{out2}) - V_{out2} \right]$$

$$V_{out2} \left[1 + \frac{1}{sT_2} + \frac{1}{s^2 T_1 T_2} \right] = \frac{V_{in}}{s^2 T_1 T_2}$$

$$H_2(s) \triangleq \frac{V_{out2}}{V_{in}} = \frac{1}{s^2 T_1 T_2 + sT_1 + 1} = \frac{1}{D(s)}$$

$$V_{out1} = \frac{V_{in}}{sT_1} [1 - H_2(s)] = \frac{V_{in}}{sT_1} \frac{sT_1 (sT_2 + 1)}{D(s)}$$

$$H_1(s) = \frac{sT_2 + 1}{D(s)}, \quad D(s) = T_1 T_2 (s^2 + s/T_2 + 1/T_1)$$

$$\omega_p = 1/\sqrt{T_1 T_2}, \quad Q_p = \omega_p T_2 = \sqrt{T_2/T_1}$$

3c.

$$D(s) \rightarrow 4T_2^2 \left[s^2 + s/T_2 + 1/(2T_2)^2 \right]$$

$$= \alpha \left(s + 1/(2T_2) \right)^2$$

Bobt poles at $-1/(2T_2) = -\frac{G_{m2}}{2C_2} = s_p$

d. $|D(j\omega_3)|^2 \propto |j\omega_3 + s_p|^4$

$$= \left[\omega_3^2 + s_p^2 \right]^2 \stackrel{!}{=} 2s_p^4$$

$$\omega_3^2 + s_p^2 \stackrel{!}{=} \sqrt{2} s_p^2$$

$$\omega_3^2 = s_p^2 (\sqrt{2} - 1) = \left(\frac{G_{m2}}{C_2} \right)^2 \frac{\sqrt{2} - 1}{4}$$

$$\omega_3 = \frac{G_{m2}}{C_2} \frac{\sqrt{\sqrt{2} - 1}}{2} \approx 0.3218 \frac{G_{m2}}{C_2}$$