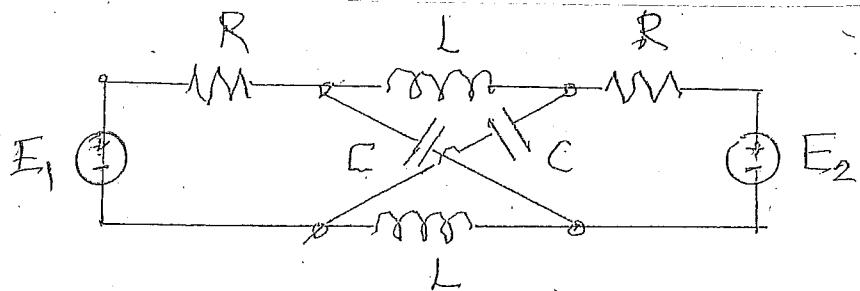


FINAL EXAMINATION
ECE 580

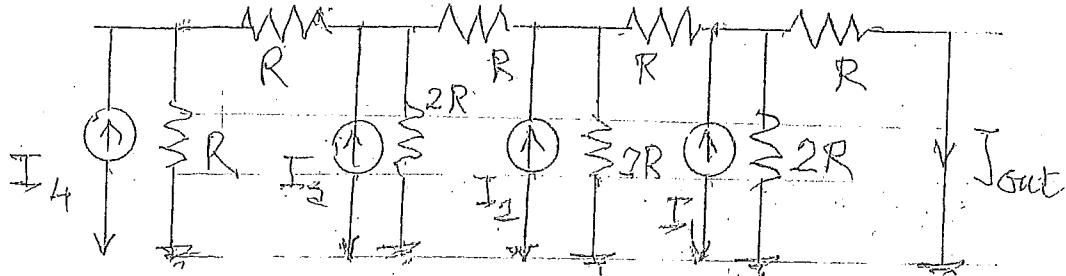
Dec. 10, 2015
Open Book

1. Find the scattering matrix S of the doubly-terminated lattice two-port shown below, at $f = 400$ MHz. The element values are $L = 10$ nH, $C = 4$ pF, $R = 50$ Ω . Carry out the following steps:

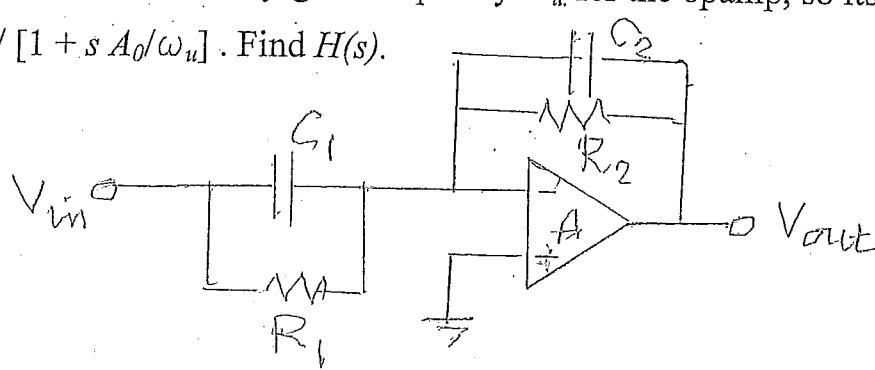
- Find Z and Z_{an} ;
- Find Y_{an} ;
- Find S .



2. Find the output current J_{out} in the ladder circuit shown. (*Hint: use interreciprocity for fast results.*)



3. a. The opamp used in the active-RC stage shown has a finite gain A_0 at all frequencies. Find the transfer function $H(s) = V_{out} / V_{in}$ of the stage.
 b. Assume next a finite unity-gain frequency ω_u for the opamp, so its gain is $A(s) = A_0 / [1 + s A_0 / \omega_u]$. Find $H(s)$.



Solution

$$1. \quad z_{11} = \frac{1}{2} (sL + \frac{1}{sC}) = z_{22}$$

$$z_{12} = z_{21} = \frac{1}{2} (\frac{1}{sC} - sL)$$

$$z_a = \begin{bmatrix} R + \frac{1}{2} (sL + \frac{1}{sC}) & \frac{1}{2} (\frac{1}{sC} - sL) \\ \frac{1}{2} (\frac{1}{sC} - sL) & R + \frac{1}{2} (sL + \frac{1}{sC}) \end{bmatrix}$$

$$z_{an} = \frac{z_a}{R} = \begin{bmatrix} 1 + \frac{1}{2R} (sL + \frac{1}{sC}) & \frac{1}{2R} (\frac{1}{sC} - sL) \\ -\frac{1}{2R} (\frac{1}{sC} - sL) & 1 + \frac{1}{2R} (sL + \frac{1}{sC}) \end{bmatrix}$$

$$Y_{an} = z_{an}^{-1} = \frac{1}{|z_{an}|} \begin{bmatrix} 1 + \frac{1}{2R} (sL + \frac{1}{sC}) & -\frac{1}{2R} (\frac{1}{sC} - sL) \\ -\frac{1}{2R} (\frac{1}{sC} - sL) & 1 + \frac{1}{2R} (sL + \frac{1}{sC}) \end{bmatrix}$$

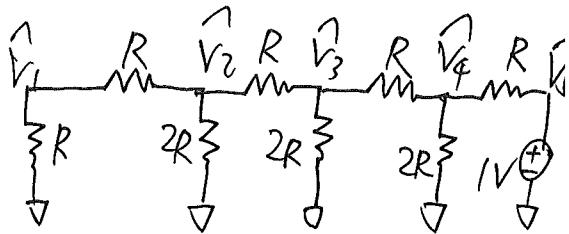
$$\text{where } |z_{an}| = \left[1 + \frac{1}{2R} (sL + \frac{1}{sC}) \right]^2 - \left[\frac{1}{2R} (\frac{1}{sC} - sL) \right]^2$$

$$= 1 + \frac{1}{R} (sL + \frac{1}{sC}) + \frac{L}{R^2 C}$$

$$\therefore S = I - 2Y_{an} = \begin{bmatrix} 1 - \frac{1}{|z_{an}|} [2 + \frac{1}{R} (sL + \frac{1}{sC})] & \frac{1}{|z_{an}|} \frac{1}{R} (\frac{1}{sC} - sL) \\ \frac{1}{|z_{an}|} \frac{1}{R} (\frac{1}{sC} - sL) & 1 - \frac{1}{|z_{an}|} [2 + \frac{1}{R} (sL + \frac{1}{sC})] \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.5966 - 0.8025j \\ 0.5966 - 0.8025j & 0 \end{bmatrix}$$

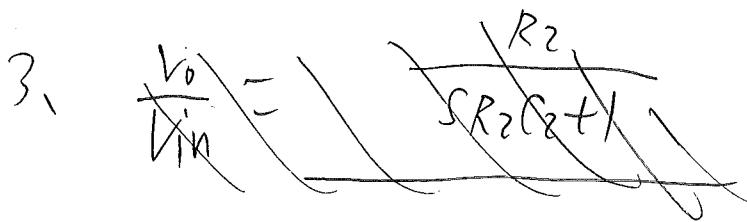
2. The adjoint network is :



Solve this network gives :

$$V_1 = \frac{1}{16}V, V_2 = \frac{1}{8}V, V_3 = \frac{1}{4}V, V_4 = \frac{1}{2}V$$

$$\begin{aligned}\therefore J_{\text{out}} &= A_1 I_1 + A_2 I_2 + A_3 I_3 + A_4 I_4 \\ &= \frac{1}{2}I_1 + \frac{1}{4}I_2 + \frac{1}{8}I_3 + \frac{1}{16}I_4\end{aligned}$$



$$\begin{aligned}
 a. \quad \frac{V_o}{V_{in}} &= -\frac{A Z_2}{A Z_1 + Z_1 + Z_2} \\
 &= -\frac{A Z_2}{(A+1) Z_1 + Z_2} \\
 &= -\frac{A \frac{R_2}{S R_2 C_2 + 1}}{(A+1) \frac{R_1}{S R_1 C_1 + 1} + \frac{R_2}{S R_2 C_2 + 1}} \\
 &= -\frac{A R_2 (S R_1 C_1 + 1)}{R_1 (A+1) \cancel{(S R_2 C_2 + 1)} + R_2 (S R_1 C_1 + 1)}
 \end{aligned}$$

b. If $A(s) = \frac{A_0}{1 + s \frac{A_0}{W_n}} = \frac{A_0 W_n}{W_n + s A_0}$

$$\begin{aligned}
 H(s) &= -\frac{\frac{A_0 W_n}{W_n + s A_0} R_2 (S R_1 C_1 + 1)}{R_1 \left(\frac{A_0 W_n}{W_n + s A_0} + 1 \right) (S R_2 C_2 + 1) + R_2 (S R_1 C_1 + 1)} \\
 &= -\frac{A_0 W_n R_2 (S R_1 C_1 + 1)}{R_1 (A_0 W_n + W_n + s A_0) (S R_2 C_2 + 1) + R_2 (W_n + s A_0) (S R_1 C_1 + 1)}
 \end{aligned}$$