ECE 580

MIDTERM EXAMINATION

Open Book

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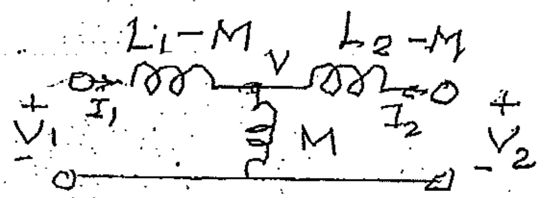
1. a. Find a T network equivalent of the physical transformer. What are the element values of the new circuit in terms of L_1, L_2 and M?

   b. Find the $ABCD$ matrix of the circuit.

   c. What is y_{11} of the transformer if it is closely coupled?

$$V_1 = \underbrace{sL_1}_{z_{11}} I_1 + \underbrace{sM}_{z_{12}} I_2$$

$$V_2 = \underbrace{sM}_{z_{21}} I_1 + \underbrace{sL_2}_{z_{22}} I_2$$



$$\underset{\sim}{T} = [ABCD] = \begin{bmatrix} z_{11}/z_{21} & |z|/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} L_1/M & \frac{s}{M}(L_1L_2 - M^2) \\ 1/sM & L_2/M \end{bmatrix}$$

($|z| = \det \underset{\sim}{z}$)

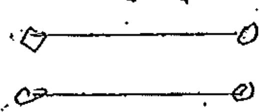
Check: $\det \underset{\sim}{T} = L_1L_2/M^2 - (L_1L_2 - M^2)/M^2 = 1$

$$y_{11} = z_{22}/|z| = sL_2 / [s^2(L_1L_2 - M^2)] = 1/[sL_1(1-k^2)]$$

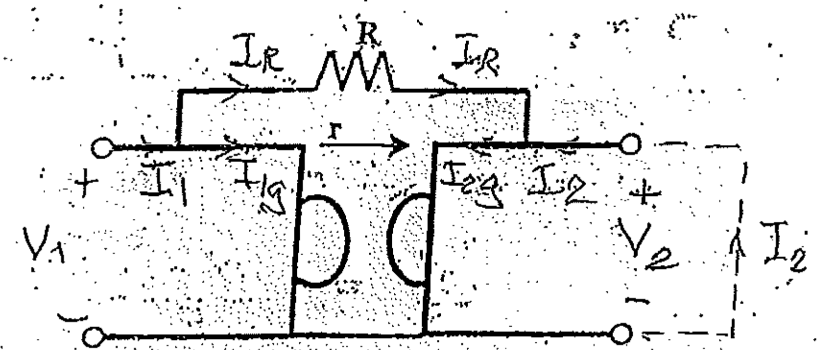
$k^2 = M^2/L_1L_2$

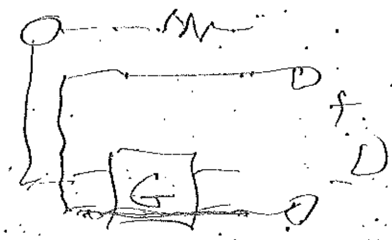
For $k \to 1$, $1/y_{11} \to 0$. All $y_{ij} \to \infty$.

The two-port becomes



2. Find the admittance matrix $\underline{Y}$ of the two-port shown below.





$$V_1 = -r I_{2g}$$

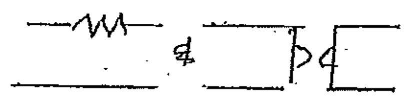
$$V_2 = r I_{1g}$$

$$y_{11} = \left(I_1/V_1\right)_{\substack{V_2=0 \\ I_{2g}=0}} = \left(I_R/V_1\right) = 1/R = y_{22}$$

$$y_{21} = \left(I_2/V_1\right)_{V_2=0} = I_{2g} - I_R = -1/r - 1/R$$

$$y_{12} = \left(y_{21}\right)_{r\to -r} = 1/r - 1/R$$

$$\underset{\sim}{Y} = \begin{bmatrix} 1/R & 1/r - 1/R \\ -1/r - 1/R & 1/R \end{bmatrix}$$

Can also break the two-port into the parallel combination of 

$$\underset{\sim}{Y_R} = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix}, \quad \underset{\sim}{Y_G} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \qquad g = 1/r,\ G = 1/R$$

$$\underset{\sim}{Y} = \underset{\sim}{Y_R} + \underset{\sim}{Y_G}$$