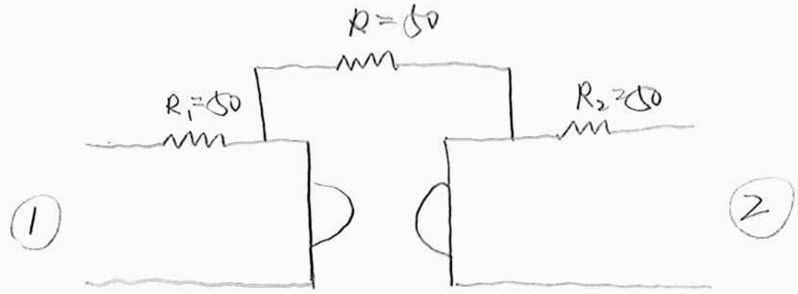
Solutions:

1. Terminated Gyrator:



a. Scattering Matrix

For the two port Gyrator without Termination

$$Y = \begin{bmatrix} \frac{1}{R} & \frac{1}{r} - \frac{1}{R} \\ -\frac{1}{r} - \frac{1}{R} & \frac{1}{R} \end{bmatrix}$$

$$\underset{=}{r=R} \begin{bmatrix} \frac{1}{R} & 0 \\ -2/R & 1/R \end{bmatrix} \Rightarrow Z = Y^{-1} = \frac{1}{\det Y} \begin{bmatrix} \frac{1}{R} & 0 \\ 2/R & 1/R \end{bmatrix}$$

$$= \begin{bmatrix} R & 0 \\ 2R & R \end{bmatrix}$$

After Terminated by R_1 & R_2,

the Augment Z matrix becomes:

$$Z_a = Z + \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} = \begin{bmatrix} R+R_1 & 0 \\ 2R & R+R_2 \end{bmatrix} = \begin{bmatrix} 2R & 0 \\ 2R & 2R \end{bmatrix}$$

$$Z_{an} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$$

Thus $Y_{an} = \frac{1}{\det Z_{an}} \begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

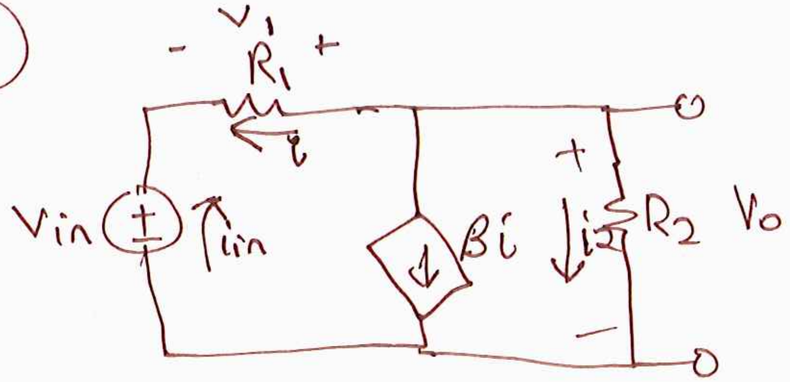
$$\Rightarrow S = 1 - 2Y_{an} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

b. $S_{21} = 1 \Rightarrow$ No Power loss inbetween the two ports

$S_{11} = S_{22} = 0 \Rightarrow$ No termination reflections

$S_{12} = 0 \Rightarrow$ Ideal reverse isolation from 2 to 1

②



$V_{in} = 5v$
$R_1 = 2K$
$R_2 = 4K$
$\beta = 4$

* Normal Circuit Parameters

by KCL

$$V_o = R_2 i_2 \qquad i_2 = -(\beta+1)i \qquad i = \frac{V_o - V_{in}}{R_1}$$

$$V_o = -\frac{R_2(\beta+1)(V_o - V_{in})}{R_1} \Rightarrow V_o\left(1 + \frac{R_2}{R_1}(\beta+1)\right) = \frac{V_{in} R_2(\beta+1)}{R_1}$$

$$V_o = \frac{V_{in} R_2(\beta+1)}{R_1\left(1 + \frac{R_2}{R_1}(\beta+1)\right)} = \frac{V_{in} R_2(\beta+1)}{R_1 + (\beta+1)}$$

* similar to voltage divider

$$R_{in} = \frac{V_{in}}{i_{in}} \qquad i_{in} = -i_1 = \frac{V_{in} - V_o}{R_1} = \frac{V_{in}}{R_1} - \frac{V_{in} R_2(\beta+1)}{R_1 + R_2(\beta+1)}$$

$$i_{in} = V_{in}\left[\frac{R_1 + R_2(\beta+1) - R_2(\beta+1)}{R_1(R_1 + R_2(\beta+1))}\right] = V_{in}\left[\frac{1}{R_1 + R_2(\beta+1)}\right]$$

$$\frac{V_{in}[R_1 + R_2(\beta+1)]}{V_{in}} \quad R_{in} = R_1 + R_2(\beta+1).$$

Senstivity for Rin due to R_1, R_2, β is very straight forward derivative

$$\boxed{\frac{\partial R_{in}}{R_1} = 1\,\frac{\Omega}{\Omega} \qquad \frac{\partial R_{in}}{\partial R_2} = \beta+1 = 5\,\frac{\Omega}{\Omega} \qquad \frac{\partial R_{in}}{\partial \beta} = R_2 = 4K\,\frac{\Omega}{unit}}$$

For first order taylor series approximation $\frac{\partial V_0}{\partial R_1}, \frac{\partial V_0}{\partial R_2}, \frac{\partial V_0}{\partial \beta}$ are still relatively easy to compute with Quotient rule

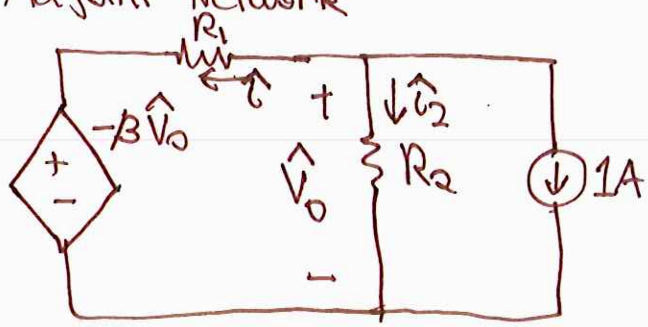
$$\boxed{\frac{\partial V_0}{\partial R_1} = \frac{0 - V_{in}(\beta+1)R_2 \cdot 1}{[R_1 + R_2(\beta+1)]^2} = \frac{-V_{in}(\beta+1)R_2}{[R_1 + R_2(\beta+1)]^2} \cong -206.6\,\frac{\mu V}{\Omega}}$$

$$\boxed{\frac{\partial V_0}{\partial R_2} = \frac{V_{in}(\beta+1)(R_1+R_2(\beta+1)) - V_{in}R_2(\beta+1)(\beta+1)}{[R_1+R_2(\beta+1)]^2} = \frac{V_{in}R_1(\beta+1)}{[R_1+R_2(\beta+1)]^2} = 103.3\,\frac{\mu V}{\Omega}}$$

$$\boxed{\frac{\partial V_0}{\partial \beta} = \frac{V_{in}R_2[R_1+R_2(\beta+1)] - V_{in}R_2(\beta+1)R_2}{[R_1+R_2(\beta+1)]^2} = \frac{V_{in}R_2R_1}{[R_1+R_2(\beta+1)]^2} \cong .0826\,\frac{V}{\text{unit}}}$$

Can also be solved using adjoint network

Adjoint Network



By Tellegen's

$$\frac{\partial V_0}{\partial R_1} = S_{R_1} = -\hat{i}\, i$$

$$\frac{\partial V_0}{\partial R_2} = S_{R_2} = -\hat{i}_2\, i_2$$

$$\frac{\partial V_0}{\partial \beta} = S_\beta = \hat{V}_0 \cdot i$$

From physical network

$$i_2 = \frac{V_0}{R_2} = \frac{50}{11.4e^3} = 1.14\,mA$$

$$i = \frac{V_0 - V_{in}}{R_1} = \left(\frac{50}{11} - 5\right)\frac{1}{2e^3} = -227\,\mu A$$

Solving Adjoint

$$\hat{V}_0 = \hat{i}_2 \cdot R_2 \quad \boxed{\hat{i}_2 = -(\hat{i}+1)}$$

$$\hat{i} = \frac{\hat{V}_0 + \beta\hat{V}_0}{R_1} = \boxed{\frac{\hat{V}_0(\beta+1)}{R_1} = \hat{i}}$$

$$\hat{V}_0 = -\left[\hat{V}_0\frac{(\beta+1)}{R_1} + 1\right]R_2 \Rightarrow \hat{V}_0\left(1 + \frac{(\beta+1)R_2}{R_1}\right) = -R_2 \Rightarrow \boxed{\hat{V}_0 = \frac{-R_2}{1 + \frac{(\beta+1)R_2}{R_1}}}$$

$\hat{V}_o = -363\tfrac{7}{11}\,V$ $\quad \hat{i} = -.909\,A$ $\quad \hat{i}_2 = -.0909\,A$

$$S_{R1} = -\hat{i}\,i = -(-.909A)(-227\mu A) = \boxed{-206\,\frac{\mu V}{\Omega} = S_{R_1}}$$

$$S_{R2} = -\hat{i}_2\,i_2 = -(-.0909A)(1.14\,mA) = \boxed{103.6\,\frac{\mu V}{\Omega} = S_{R2}}$$

$$S_\beta = \hat{V}_o \cdot i = \left(-363\tfrac{7}{11}V\right)\left(-227\mu A\right) = \boxed{.0825 = S_\beta\ [V/\text{unit}]}$$

A Results between two methods are closely matched