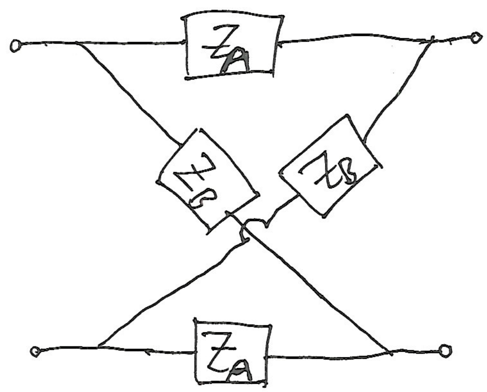
HW 2 Solution

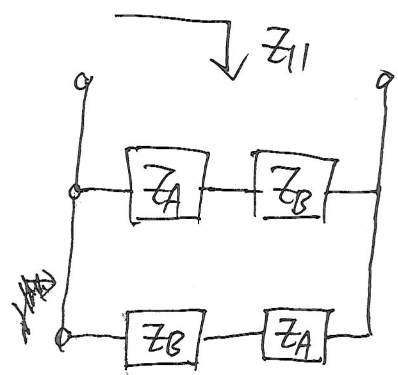
1. (a)



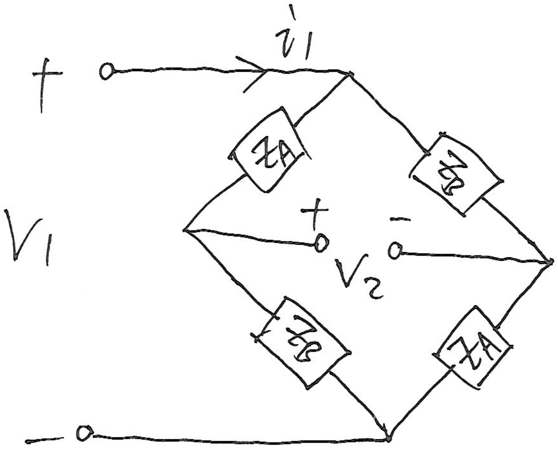
$$Z_A = s + \frac{1}{\frac{1}{s}+s} = \frac{s^3+2s}{s^2+1}$$

$$Z_B = s + \frac{1}{s} = \frac{s^2+1}{s}$$

$$z_{11} = \frac{Z_A+Z_B}{2} = \frac{s^4+2s^2+\frac{1}{2}}{s^3+s}$$



To get z_{21}, use



$$V_2 = V_1 \cdot \frac{Z_B}{Z_A + Z_B} - V_1 \cdot \frac{Z_A}{Z_A + Z_B}$$

$$= V_1 \cdot \frac{Z_B - Z_A}{Z_A + Z_B}$$

$$I_1 = \frac{V_1}{\frac{Z_A + Z_B}{2}} = \frac{2V_1}{Z_A + Z_B}$$

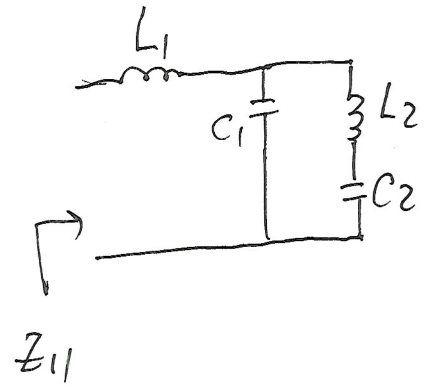
$$\therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{V_1 \frac{Z_B - Z_A}{Z_A + Z_B}}{\frac{2V_1}{Z_A + Z_B}} = \frac{Z_B - Z_A}{2}$$

$$= \frac{1}{2s(s^2+1)}$$

Since it is symmetric, $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$

$$\therefore Z = \begin{bmatrix} \frac{s^4 + 2s^2 + \frac{1}{2}}{s^3 + s} & \frac{1}{2s(s^2+1)} \\ \frac{1}{2s(s^2+1)} & \frac{s^4 + 2s^2 + \frac{1}{2}}{s^3 + s} \end{bmatrix}$$

(b)



$$Z_{11} = sL_1 + \frac{1}{sC_1} // \left(sL_2 + \frac{1}{sC_2}\right)$$

$$= \frac{s^4 C_1 C_2 L_2 L_1 + s^2 (C_1 L_1 + C_2 L_1 + C_2 L_2) + 1}{s^3 C_1 C_2 L_2 + s(C_1 + C_2)}$$

$$= \frac{s^4 + 2s^2 + \frac{1}{2}}{s^3 + s} = \frac{2s^4 + 4s^2 + 1}{2s^3 + 2s}$$

Also, $C_1 = C_2$, $L_1 = L_3$

$$\therefore \begin{cases} C_1 C_2 L_1 L_2 = 2 \\ C_1 L_1 + C_2 L_1 + C_2 L_2 = 4 \\ C_1 C_2 L_2 = 2 \\ C_1 + C_2 = 2 \end{cases} \Longrightarrow \begin{cases} L_1 = 1\,H = L_3 \\ L_2 = 2\,H \\ C_1 = 1\,F \\ C_2 = 1\,F \end{cases}$$

Use Z_{21} should get same results.

(c). From Page 63 of lecture notes,

$$A_v = \frac{V_2}{V_1} = \frac{Z_{12}}{Z_{11}} = \frac{1}{2s^4 + 4s^2 + 1}$$

A transmission zero is a frequency where $A_v = 0$.

Since A_v is never zero, such zero doesn't exist.