This result indicates that the center frequency ω_0 is reduced, and the quality factor has increased over its enhanced value Q in Eq. (4.140); the *realized* parameters are

$$\omega_{0r} \approx \frac{\omega_0}{\sqrt{1+\varepsilon}}, \quad Q_r = \frac{Q_0}{1-2Q_0^2\alpha}\sqrt{1+\varepsilon} = Q\sqrt{1+\varepsilon} \tag{4.152}$$

and the *realized* gain is approximately unaffected by the opamp, $H_r = H_B$ of Eq. (4.141). For instance, for the design parameters in Example 4.10 we have from Eq. (4.150)

$$\varepsilon = \frac{2Q_0}{|A(j\omega_0)|}\frac{1}{(1-K)^2} = \frac{3}{1500/12.5}\frac{1}{(1-0.159)^2} = 0.035$$

that is, the frequency error equals -1.7% and the Q error is $+1.7\%$. In the Delyiannis–Friend circuit with no Q enhancement, we had $K = 0$ and $Q_0 = Q = 10$; the error then becomes

$$\varepsilon = \frac{2Q}{|A(j\omega_0)|} = \frac{20}{1500/12.5} = 0.17$$

and the errors would be equal to -8% and $+8\%$ for frequency and Q, respectively, as was observed in Example 4.9. We note, therefore, that Q enhancement brings two notable advantages for the small price of two resistors. The component ratio is substantially reduced (from 400 in Example 4.9 to nine in Example 4.10) and the errors in frequency and quality factor are significantly smaller (by about a factor of five in these examples). Whenever the use of power, considerations of space, and cost are of prime concern, the single-amplifier Q-enhanced Delyiannis–Friend biquad is generally the most versatile and least sensitive option. For cases of medium and high Q, we will therefore in the remainder of the book mostly use the circuit in Fig. 4.37 when a single-amplifier biquad is needed.

4.5.3 Rauch Filters

Another popular SAB (Single-Amplifier Biquad) multiple-feedback circuit is the so-called *Rauch* filter (*Radio Telemetry* by L. L. Rauch, 1956). It is shown in Fig. 4.39 in its lowpass configuration. Observe that this circuit has the same topology as the Delyiannis–Friend (Fig. 4.35) circuits, but the Rauch filter places resistors and capacitors in different circuit

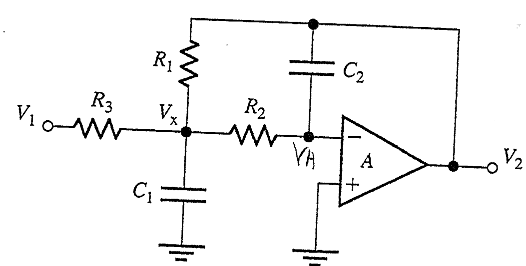

Figure 4.39 A lowpass Rauch filter section.

branches and has been found particularly convenient for building lowpass functions. Due to its SAB structure, it should be clear that by choosing different configurations for the passive elements, bandpass or highpass filters can also be built; indeed, the Rauch filter can be configured to realize finite transmission zeros by feeding V_1 directly to the inverting opamp input, V_-, through an additional RC admittance. However, Rauch circuits have been applied most widely as lowpass filters used as band-limiting, anti-aliasing, and reconstruction filters for digital and sampled-data systems, and we shall limit our discussion to all-pole lowpass filters. The circuit has low sensitivities to component tolerances (see Chapter 12), is easy to adjust, is absolutely stable, and, as all single-opamp filters, is efficient in its use of space and power. To analyze the circuit's performance, we start as usual with the node equation at the inverting opamp input node:

$$V_-\,(sC_2 + G_2) = sC_2V_2 + G_2V_x$$

With $V_- = -V_2/A$ this equation becomes

$$V_2\left(sC_2 + \frac{sC_2 + G_2}{A}\right) = -G_2V_x \tag{4.153}$$

At the node labeled V_x we sum the currents to get

$$V_x\,(G_1 + G_2 + G_3 + sC_1) = G_3V_1 + G_1V_2 - G_2V_2/A \tag{4.154}$$

and combining these two equations results in

$$V_2\left[\left(sC_2R_2 + \frac{sC_2R_2 + 1}{A}\right)(G_1 + G_2 + G_3 + sC_1) + G_1 - \frac{G_2}{A}\right] = -G_3V_1$$

that is, the transfer function is

$$\frac{V_2}{V_1} = -\frac{G_2G_3/(C_1C_2)}{s^2 + s\,(G_1 + G_2 + G_3)/C_1 + G_1G_2/(C_1C_2) + \varepsilon} \tag{4.155}$$

where

$$\varepsilon = \frac{1}{A}\left[s^2 + s\left(\frac{G_1 + G_2 + G_3}{C_1} + \frac{G_2}{C_2}\right) + \frac{(G_1 + G_3)\,G_2}{C_1C_2}\right] \tag{4.156}$$

is the error term caused by the finite opamp gain.

Let us for now assume ideal opamps. Evidently, then, with $A = \infty$ we get $\varepsilon = 0$ and obtain from Eq. (4.155) dc gain, pole frequency, and quality factor as

$$H_0 = \frac{G_3}{G_1} \qquad \omega_0 = \sqrt{\frac{G_1G_2}{C_1C_2}} \tag{4.157a, b}$$

$$Q = \frac{\sqrt{C_1/C_2}}{\sqrt{G_1/G_2} + \sqrt{G_2/G_1} + G_3/\sqrt{G_1G_2}} = \frac{\sqrt{C_1/C_2}}{\sqrt{G_1/G_2}\,(1 + H_0) + \sqrt{G_2/G_1}} \tag{4.157c}$$

Note that the transfer function is inverting; but we have defined the gain, H_0, as a positive resistor ratio so that the minus sign will not permeate the equations. We observe again that Q is completely determined by ratios of like elements, but with no difference effects, so that Q can be designed very accurately. For positive elements Q is always positive, i.e., the Rauch lowpass is unconditionally stable, in contrast to, e.g., the Sallen–Key circuit, where Q is set by the difference of component ratios, Eq. (4.103), and can become negative if the element values are inaccurate. Note that Q is usually small for lowpass filters; therefore, no excessively high element ratios will be required, and there is no need to enhance Q through carefully selected positive feedback as in the Delyiannis–Friend circuit. Clearly, by Eq. (4.157c) the capacitor ratio is proportional to Q^2. Therefore, the Rauch circuit can generally not be recommended for filters with large values of Q because it leads to an excessive capacitor spread. The Rauch lowpass has five passive elements, but we have in a lowpass only three parameters: pole frequency ω_0, quality factor Q, and gain H_0. Apparently this leaves us with two free components that can be determined from other constraints. However, note that by Eq. (4.157c) all five components together determine Q.

Although not critical for low-Q lowpass filters, let us nevertheless minimize the capacitor ratio, C_1/C_2: for given values of H_0 and Q we reformat Eq. (4.157c) into

$$\frac{C_1}{C_2} = Q^2 \left[\sqrt{\frac{G_1}{G_2}}\left(1 + \frac{G_2}{G_1} + H_0\right)\right]^2$$
$$= Q^2 \left[\sqrt{1+H_0}\left(\sqrt{\frac{(1+H_0)\,G_1}{G_2}} + \sqrt{\frac{G_2}{(1+H_0)\,G_1}}\right)\right]^2 \qquad \textbf{(4.158)}$$

and recognize that the term in brackets with $H_0 = G_3/G_1$ is of the form $\sqrt{1+H_0}\,(x + 1/x)$ with a minimum of $2\sqrt{1+H_0}$ at $x = 1$. Clearly, then, if the minimum capacitor spread is important, the choice is

$$\frac{G_2}{(1+H_0)\,G_1} = 1, \text{ i.e., } R_1 = (1+H_0)\,R_2 \qquad \textbf{(4.159)}$$

Consequently, the lowest possible value of the capacitor ratio is $C_1/C_2 = 4Q^2\,(1+H_0)$, i.e., for low-Q lowpass filters, where H_0 is near 1, $C_1/C_2 \approx 8Q^2$. If we were to choose simply all resistors of equal value, the dc gain would be fixed at $H_0 = 1$ and, by Eq. (4.158), the ratio would be $C_1/C_2 = 9Q^2$, only a small penalty.

To arrive at the component values from Eq. (4.157), a suitable method is then via the following five steps:

1. Choose R_1 and set $R_3 = R_1 H_0$ to realize H_0. A suitable choice of R_1 is in the same range, say kiloohms, where all the resistors are expected to be.

2. Then set $R_2 = R_1/(1+H_0)$ by Eq. (4.159) to minimize the capacitor ratio, or simply choose $R_2 = R_1$.

3. Determine the *capacitor product*, $C_1C_2 = 1/\left(\omega_0^2 R_2 R_1\right)$, to set ω_0.

4. Determine the *ratio* $C_1/C_2 = Q^2\left[\sqrt{R_2/R_1}\,(1+H_0) + \sqrt{R_1/R_2}\right]^2$ to set Q.

5. The individual capacitors are found by multiplying and dividing, respectively, the product $C_1 C_2$ and the ratio C_1/C_2.

In low-Q lowpass filters the errors in ω_0 and Q arising from finite values of ω_t of real opamps usually cause no problems; nevertheless, to get an understanding of the magnitudes to be expected, let us analyze the effects.

The Effect of A(s) on the Rauch Lowpass Filter

We saw in Eq. (4.155) that the denominator of the transfer function contains an additive term ε, Eq. (4.156), which with $H_0 = G_3/G_1$ and $(G_1 + G_2 + G_3)/C_1 = \omega_0/Q$ is seen to equal

$$\varepsilon = \frac{1}{A}\left[s^2 + s\left(\frac{\omega_0}{Q} + \frac{1}{R_2 C_2}\right) + \omega_0^2 (1 + H_0)\right] \qquad \textbf{(4.160)}$$

so that the transfer function including the error can be written as

$$\begin{aligned}\frac{V_2}{V_1} &= -\frac{A}{1+A}\frac{G_2 G_3/(C_1 C_2)}{\left(s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2\right) + \dfrac{1}{1+A}\dfrac{G_2}{C_2}\left(s + \dfrac{G_1}{C_1}H_0\right)}\\ &= -\frac{A}{1+A}\frac{\omega_0^2 H_0}{s^2 + s\dfrac{\omega_0}{Q}\left(1 + \dfrac{Q}{1+A}\dfrac{G_2}{\omega_0 C_2}\right) + \omega_0^2\left(1 + \dfrac{H_0}{1+A}\right)} \qquad \textbf{(4.161}\end{aligned}$$

Using the approximation $A = \omega_t/s$, Eq. (2.18), where for a lowpass filter $|s| \ll \omega_t$, we ha $1/(A+1) = s/(s+\omega_t) \approx s/\omega_t$ and $A/(A+1) = \omega_t/(s+\omega_t) \approx 1$, so Eq. (4.161) becomes

$$\begin{aligned}\frac{V_2}{V_1} &= -1 \times \frac{\omega_0^2 H_0}{s^2\left(1 + \dfrac{1}{\omega_t}\dfrac{G_2}{C_2}\right) + s\dfrac{\omega_0}{Q}\left(1 + \dfrac{\omega_0}{\omega_t}QH_0\right) + \omega_0^2}\\ &\approx -\frac{\omega_0^2 H_0/[1 + G_2/(\omega_t C_2)]}{s^2 + s\dfrac{\omega_0}{Q}\dfrac{1 + QH_0\omega_0/\omega_t}{1 + G_2/(\omega_t C_2)} + \dfrac{\omega_0^2}{1 + G_2/(\omega_t C_2)}}\\ &= \frac{H_0\omega_r^2}{s^2 + s\omega_r/Q_r + \omega_r^2} \qquad \textbf{(4.162)}\end{aligned}$$

Thus, within the approximations made, A does not affect the dc gain. The realized pole frequency, ω_r, relative to the desired ω_0 for $G_2/(\omega_t C_2) \ll 1$, is given by

$$\frac{\omega_r}{\omega_0} = \frac{1}{\sqrt{1 + G_2/(\omega_t C_2)}} \approx 1 - \frac{1}{2}\frac{G_2}{C_2\omega_0}\frac{\omega_0}{\omega_t} = 1 - \frac{\omega_0 C_1}{2G_1}\frac{\omega_0}{\omega_t} \qquad \textbf{(4.163)}$$

$$H(s) = \frac{H(s) + H(-s)}{2} + \frac{H(s) - H(-s)}{2}$$

$$= H_e(s) + H_o(s), \quad \begin{cases} H_e(s) = H_e(-s) & \text{even} \\ H_o(-s) = -H_o(s) & \text{odd} \end{cases}$$

~~12-39~~ 3 (easy)

Since $H(s)$ is real rational, it can be written separately in even part and odd part

$$H(s) = H_e(s) + H_o(s) = P_e(s) + s\,Q_e(s)$$

$$= R_1(s^2) + s\,R_2(s^2)$$

For $s = j\omega$

$$H(j\omega) = R_1(-\omega^2) + j\omega\,R_2(-\omega^2)$$

$$\beta(\omega) = \tan^{-1} \frac{\omega R_2(-\omega^2)}{R_1(-\omega^2)} = \tan^{-1} U(\omega)$$

$U(\omega) = \dfrac{\omega R_2(-\omega^2)}{R_1(-\omega^2)}$ is an odd function.

Also $\theta(x) = \tan^{-1} x$ is an odd function.

$\beta(\omega)$ is obviously an odd function;

i.e. $\beta(-\omega) = -\beta(\omega)$

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