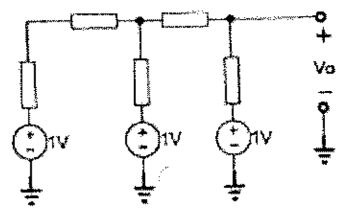
Midterm Examination

ECE 580

October 20, 2021

Open book

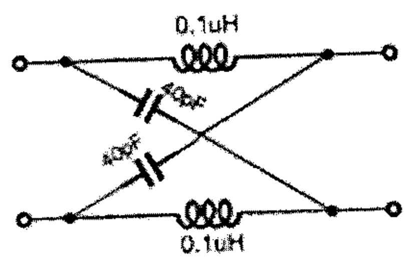
1. Find the branch currents and the output voltage in this ladder. All impedances are 1 kΩ. Hint: think carefully before starting to calculate!



2. The two-port shown operates between two 50 Ω resistors, at ω = 500 Mra/s.

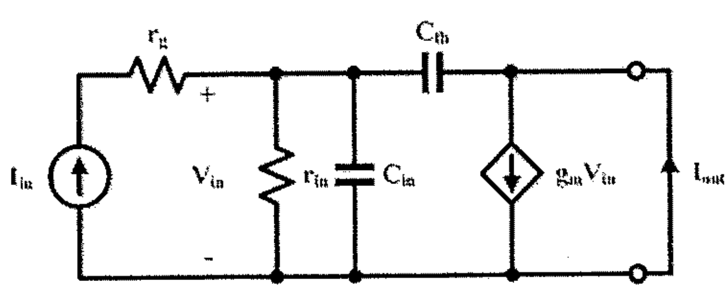
   (a) Find its scattering matrix.

   (b) What are the properties of the circuit? What role does it play?



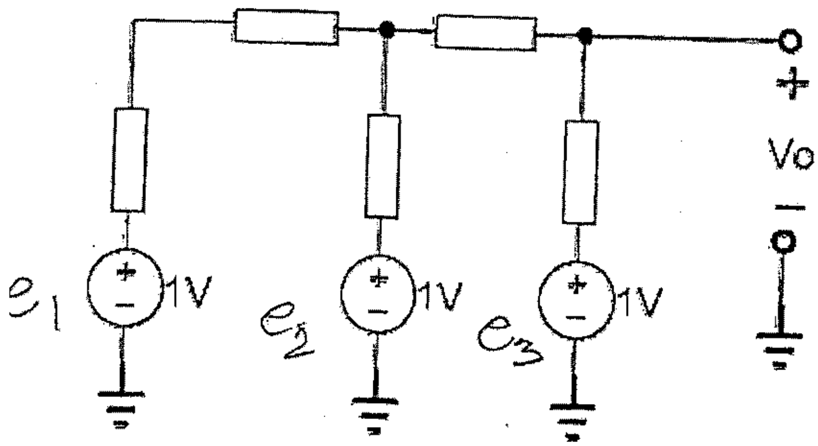
3.(a) Find the current gain $A_I(j\omega)$ of the circuit shown.

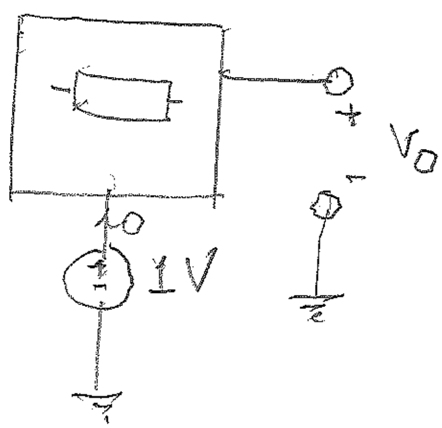
(b) Find the radian frequency ω_1 where $|A_I(j\omega_1)| = 1$.



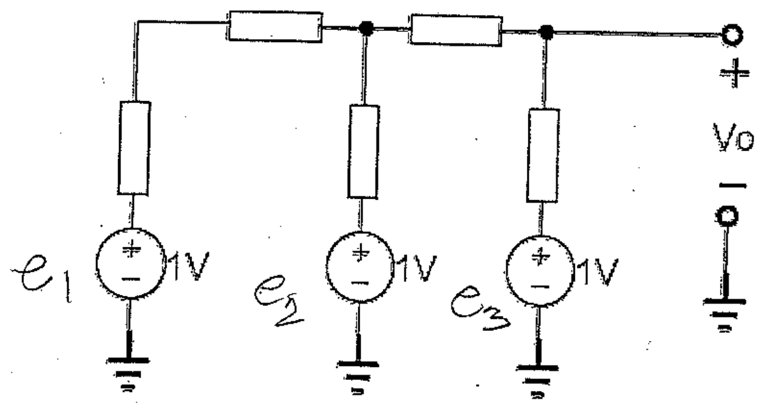
ECE 580 Midterm Examination Solutions

1. Since $e_1 = e_2 = e_3$, all loop voltages are zero. No currents flow, and all node voltages including v_0 are 1 V. Can also be seen by connecting the + terminals of the voltage sources.

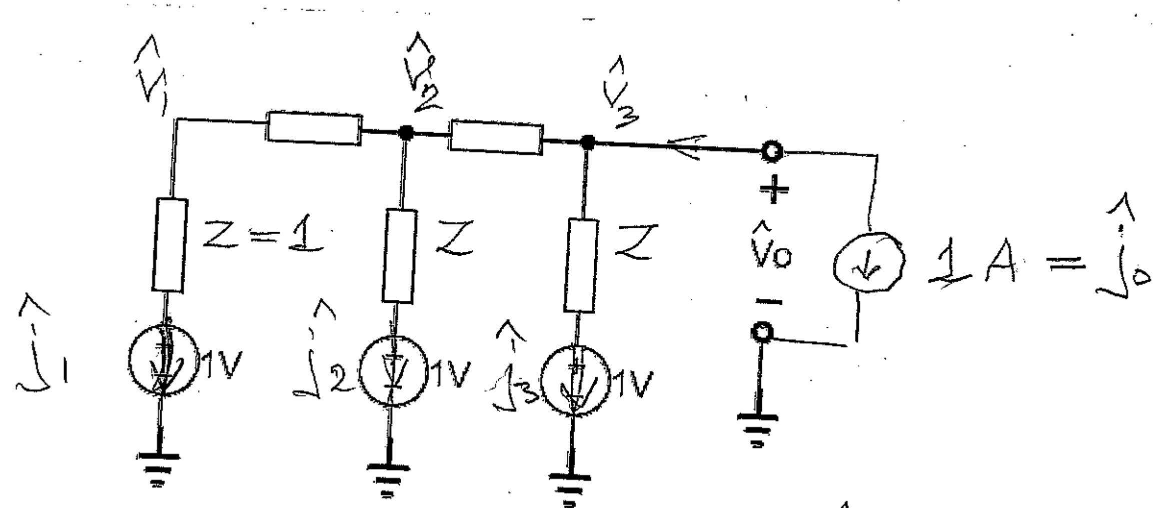




1. Find the output voltage of the circuit shown using interreciprocity. All impedances are equal.



$\hat{N}$ is



Assume $\hat{j}_1 = 1A$. Then $\hat{v}_1 = 1$, $\hat{v}_2 = 2$

$\hat{j}_2 = 2$, $\hat{v}_3 = \hat{j}_1 + \hat{j}_2 + \hat{v}_2 = 5$

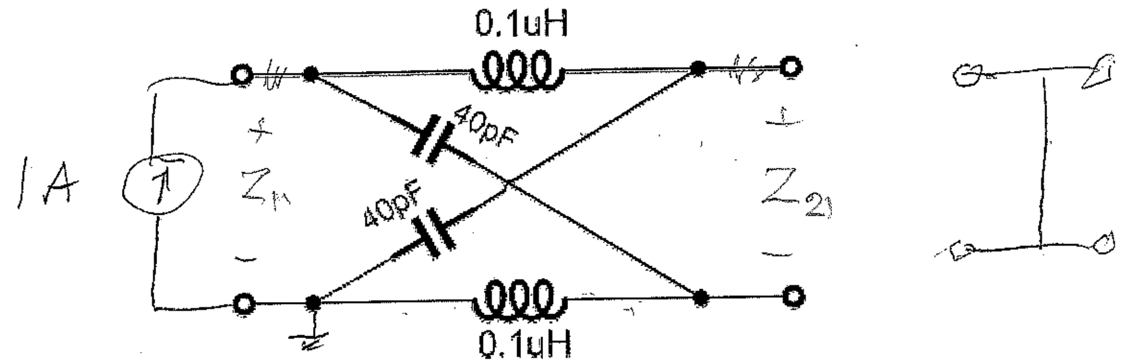
$\hat{j}_3 = 5$, $-\hat{j}_0 = \hat{j}_1 + \hat{j}_2 + \hat{j}_3 = 8$

Since $\hat{j}_0 = +1$, the actual currents are

$\hat{j}_1 = +1/8$, $\hat{j}_2 = -1/4$, $\hat{j}_3 = -5/8$

$V_0 = -\sum_{i=1}^{3} e_i \hat{j}_i = 1\,V$. Predictable, since

2. The two-port shown operates between two 50 ohm terminations at a frequency $\omega = 500$ Mra/s. Find its scattering matrix.



$$Z_L = j5\times10^8 \times 10^{-7} = j50\,\Omega$$

$$Z_C = -j/(5\times10^8 \times 4\times10^{-11}) = -j50\,\Omega$$

$$Z_{11} = Z_{22} = \frac{1}{2}(Z_L + Z_C) = 0$$

$$Z_{12} = Z_{21} = \frac{1}{2}[Z_C - Z_L] = -j50\,\Omega$$

$$\underset{\sim}{Z} = \begin{bmatrix} 0 & -j50 \\ -j50 & 0 \end{bmatrix}, \quad \underset{\sim}{Z_a} = 50\begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

$$\underset{\sim}{Z_{an}} = \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}, \quad \underset{\sim}{Y_{an}} = \frac{1}{2}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

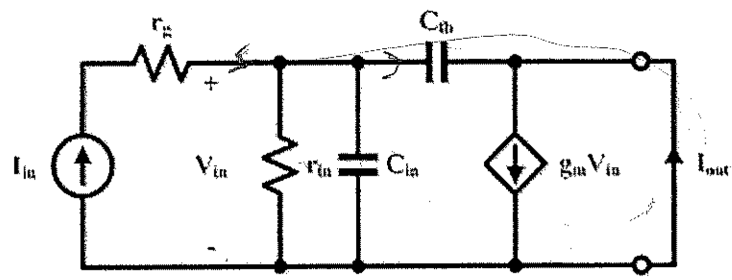
$$\underset{\sim}{S} = \underset{\sim}{1} - 2\underset{\sim}{Y_{an}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$\underset{\sim}{S} = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \quad -90^\circ \text{ phase shifter}$$

Lossless

3.(a) Find the current gain $A_I(j\omega)$ of the circuit shown.

(b) Find the radian frequency ω_1 where $|A_I(j\omega_1)| = 1$. unity-gain fr.



$$V_{in} = I_{in}\left(\frac{1}{r_{in}} + s(C_{in} + C_{fb})\right)^{-1}$$

$$I_{out} = g_m V_{in} - sC_{fb} V_{in}$$

$$= I_{in}(g_m - sC_{fb})\left[1/r_{in} + s(C_{in} + C_{fb})\right]^{-1}$$

$$A_I = \frac{g_m - j\omega C_{fb}}{1/r_{in} + j\omega(C_{in} + C_{fb})} \qquad s = j\omega$$

$$|A_I|^2 = \frac{g_m^2 + \omega^2 C_{fb}^2}{1/r_{in}^2 + \omega^2 (C_{in} + C_{fb})^2} \rightarrow 1$$

$$g_m^2 - 1/r_{in}^2 \stackrel{!}{=} \omega_T^2\left[(C_{in} + C_{fb})^2 - C_{fb}^2\right]$$

$$\omega_T^2 = \left[\frac{g_m^2 - 1/r_{in}^2}{C_{in}^2 + 2C_{in}C_{fb}}\right]$$