This result indicates that the center frequency ω_0 is reduced, and the quality factor has increased over its enhanced value Q in Eq. (4.140); the *realized* parameters are

$$\omega_{0r} \approx \frac{\omega_0}{\sqrt{1+\varepsilon}}, \quad Q_r = \frac{Q_0}{1-2Q_0^2\alpha}\sqrt{1+\varepsilon} = Q\sqrt{1+\varepsilon} \tag{4.152}$$

and the *realized* gain is approximately unaffected by the opamp, $H_r = H_B$ of Eq. (4.141). For instance, for the design parameters in Example 4.10 we have from Eq. (4.150)

$$\varepsilon = \frac{2Q_0}{|A(j\omega_0)|}\frac{1}{(1-K)^2} = \frac{3}{1500/12.5}\frac{1}{(1-0.159)^2} = 0.035$$

that is, the frequency error equals -1.7% and the Q error is $+1.7\%$. In the Delyiannis–Friend circuit with no Q enhancement, we had $K = 0$ and $Q_0 = Q = 10$; the error then becomes

$$\varepsilon = \frac{2Q}{|A(j\omega_0)|} = \frac{20}{1500/12.5} = 0.17$$

and the errors would be equal to -8% and $+8\%$ for frequency and Q, respectively, as was observed in Example 4.9. We note, therefore, that Q enhancement brings two notable advantages for the small price of two resistors. The component ratio is substantially reduced (from 400 in Example 4.9 to nine in Example 4.10) and the errors in frequency and quality factor are significantly smaller (by about a factor of five in these examples). Whenever the use of power, considerations of space, and cost are of prime concern, the single-amplifier Q-enhanced Delyiannis–Friend biquad is generally the most versatile and least sensitive option. For cases of medium and high Q, we will therefore in the remainder of the book mostly use the circuit in Fig. 4.37 when a single-amplifier biquad is needed.

4.5.3 Rauch Filters

Another popular SAB (Single-Amplifier Biquad) multiple-feedback circuit is the so-called *Rauch* filter (*Radio Telemetry* by L. L. Rauch, 1956). It is shown in Fig. 4.39 in its lowpass configuration. Observe that this circuit has the same topology as the Delyiannis–Friend (Fig. 4.35) circuits, but the Rauch filter places resistors and capacitors in different circuit

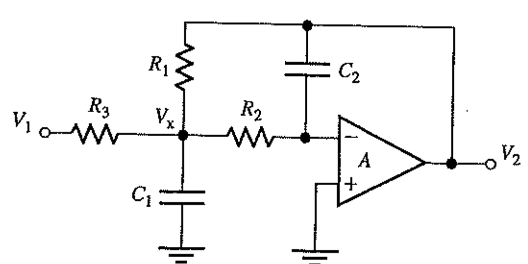

Figure 4.39 A lowpass Rauch filter section.

branches and has been found particularly convenient for building lowpass functions. Due to its SAB structure, it should be clear that by choosing different configurations for the passive elements, bandpass or highpass filters can also be built; indeed, the Rauch filter can be configured to realize finite transmission zeros by feeding V_1 directly to the inverting opamp input, V_-, through an additional RC admittance. However, Rauch circuits have been applied most widely as lowpass filters used as band-limiting, anti-aliasing, and reconstruction filters for digital and sampled-data systems, and we shall limit our discussion to all-pole lowpass filters. The circuit has low sensitivities to component tolerances (see Chapter 12), is easy to adjust, is absolutely stable, and, as all single-opamp filters, is efficient in its use of space and power. To analyze the circuit's performance, we start as usual with the node equation at the inverting opamp input node:

$$V_-\left(sC_2 + G_2\right) = sC_2V_2 + G_2V_x$$

With $V_- = -V_2/A$ this equation becomes

$$V_2\left(sC_2 + \frac{sC_2 + G_2}{A}\right) = -G_2V_x \tag{4.153}$$

At the node labeled V_x we sum the currents to get

$$V_x\left(G_1 + G_2 + G_3 + sC_1\right) = G_3V_1 + G_1V_2 - G_2V_2/A \tag{4.154}$$

and combining these two equations results in

$$V_2\left[\left(sC_2R_2 + \frac{sC_2R_2 + 1}{A}\right)\left(G_1 + G_2 + G_3 + sC_1\right) + G_1 - \frac{G_2}{A}\right] = -G_3V_1$$

that is, the transfer function is

$$\frac{V_2}{V_1} = -\frac{G_2G_3/(C_1C_2)}{s^2 + s\left(G_1 + G_2 + G_3\right)/C_1 + G_1G_2/(C_1C_2) + \varepsilon} \tag{4.155}$$

where

$$\varepsilon = \frac{1}{A}\left[s^2 + s\left(\frac{G_1 + G_2 + G_3}{C_1} + \frac{G_2}{C_2}\right) + \frac{\left(G_1 + G_3\right)G_2}{C_1C_2}\right] \tag{4.156}$$

is the error term caused by the finite opamp gain.

Let us for now assume ideal opamps. Evidently, then, with $A = \infty$ we get $\varepsilon = 0$ and obtain from Eq. (4.155) dc gain, pole frequency, and quality factor as

$$H_0 = \frac{G_3}{G_1} \qquad \omega_0 = \sqrt{\frac{G_1G_2}{C_1C_2}} \tag{4.157a, b}$$

$$Q = \frac{\sqrt{C_1/C_2}}{\sqrt{G_1/G_2} + \sqrt{G_2/G_1} + G_3/\sqrt{G_1G_2}} = \frac{\sqrt{C_1/C_2}}{\sqrt{G_1/G_2}\left(1 + H_0\right) + \sqrt{G_2/G_1}} \tag{4.157c}$$

As mentioned earlier, the main reason for the dip and the subsequent rise in gain at high frequencies of the lowpass response is the output resistance of the opamp. Note that the gain at $s = j\omega_0$ is $T(j\omega_0) = KQ = 14.06$, or 23 dB. We mentioned earlier that the EWB "measurement" is actually based on a SPICE simulation; we have shown in Fig 4.34b the SPICE results to be able to display directly both V_{LP} and V_{HP} simultaneously.

Since the *RC–CR* transformation did not change the filter poles, the earlier warning concerning the sensitivity of Q to the gain K is still valid, as are the discussion and results on the effect of finite opamp gain. Observe further that if a highpass with smaller gain is required, a procedure analogous to the one used for the lowpass in Fig. 4.31b can be used. The difference is that now we have a capacitor in series with the input so that we obtain a capacitive voltage divider with elements aC_1 and $(1-a)C$. For example, if in the previous example a high-frequency gain of 0 dB is specified, i.e., $a = 1/K = 0.357$, the $C = 0.1$-μF capacitor will have to be split into a series capacitor of 35.7 nF and a shunt capacitor of 64.3 nF.

4.5.2 The Single-Amplifier Biquad (SAB)

A useful bandpass circuit using only one operational amplifier was developed by T. Delyiannis (1968) and J. J. Friend (1970). It is shown in its bandpass configuration in Fig. 4.35. Both capacitors are labeled C because these filters are normally built with identical capacitors. We can determine the circuit's operation by writing a node equation for the inverting input terminal of the opamp and one for the node labeled V_x. The two equations are

$$(2sC + G_1)\,V_x = aG_1V_1 + sCV_2 + sCV_- \tag{4.124}$$

$$(sC + G_2)\,V_- = sCV_x + G_2V_2 \tag{4.125}$$

with

$$V_- = -V_2/A \tag{4.126}$$

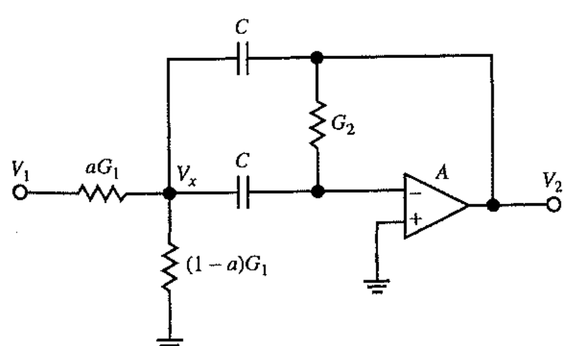

Figure 4.35 Delyiannis–Friend bandpass circuit.

We have again assumed finite opamp gain A to be able to investigate later which effect A may have on filter parameters with no need to repeat the analysis. For an ideal opamp, $V_- = 0$ and the terms multiplied by V_- will simply be absent. To solve these equations for the transfer function V_2/V_1, we solve Eq. (4.125) for V_x and insert the result into Eq. (4.124). We find

$$V_x = -\frac{1}{sC}\left(G_2 + \frac{sC + G_2}{A}\right)V_2$$

and

$$V_2\left[\frac{1}{sC}\left(G_2 + \frac{sC + G_2}{A}\right)(2sC + G_1) + sC - \frac{sC}{A}\right] = -aG_1V_1$$

This equation gives us the transfer function as

$$T(s) = \frac{V_2}{V_1} = \frac{N(s)}{D(s)} = -\left(\frac{A}{1+A}\right)\frac{saG_1/C}{s^2 + s\left(\frac{2G_2}{C} + \frac{G_1/C}{1+A}\right) + \frac{G_1G_2}{C^2}} \tag{4.127}$$

The parameter a to set the gain is obtained by a feed-in voltage divider in the same manner as for the Sallen–Key circuit in Fig. 4.31b. We observe that the first effect of finite opamp gain is the multiplying factor $A/(1+A)$. This is simply the gain of a voltage buffer, Eq. (2.82), and can be neglected for frequencies less than the opamp's unity-gain frequency, $f < f_t$. The second effect is determined by the term $(G_1/C)/(1+A)$ in the denominator. We shall investigate its consequences later and assume for now an ideal opamp, i.e., we consider the ideal transfer function

$$T(s) = -\frac{saCG_1}{s^2C^2 + 2sCG_2 + G_1G_2} = -\frac{saG_1/C}{s^2 + s2G_2/C + G_1G_2/C^2} \tag{4.128}$$

Let us rewrite Eq. (4.128) in the standard form with center frequency ω_0 and quality factor Q_0,

$$T(s) = \frac{V_2}{V_1} = -\frac{sH\,(\omega_0/Q_0)}{s^2 + s\,(\omega_0/Q_0) + \omega_0^2} \tag{4.129}$$

We can then identify the filter parameters as

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C^2}}, \quad Q_0 = \frac{1}{2}\omega_0CR_2 = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}, \quad \text{and} \quad H = \frac{1}{2}a\frac{R_2}{R_1} = 2aQ_0^2 \tag{4.130}$$

Conversely, we can also express the element values directly in terms of the filter parameters. From Eq. (4.130) we find the design equations as follows:

$$R_1R_2 = \frac{1}{\omega_0^2C^2}, \quad \frac{R_2}{R_1} = 4Q_0^2, \quad \text{and} \quad a = \frac{H}{2Q_0^2} \tag{4.131}$$

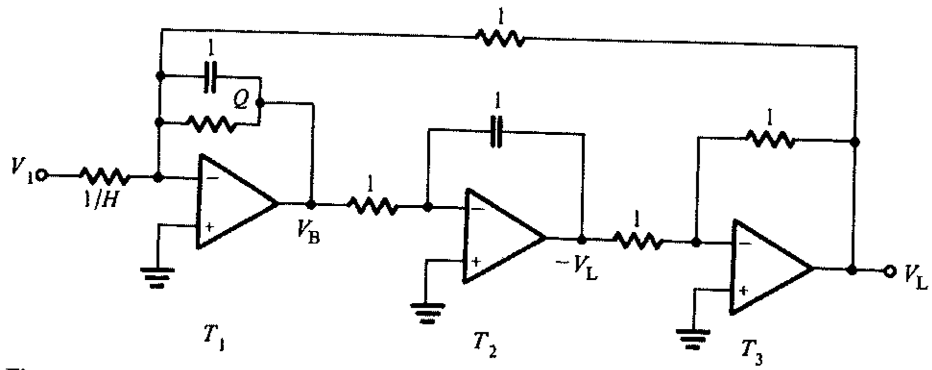

Figure 4.9 The Tow–Thomas biquad with normalized elements.

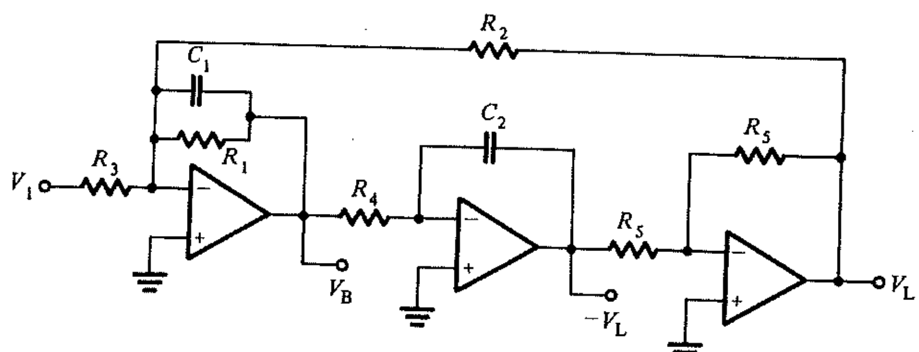

Figure 4.10 The Tow–Thomas biquad.

and making the connections suggested by the labels at the terminals of the three modules results in the full circuit in Fig. 4.9. This filter is the so-called *Tow–Thomas* biquad (Tow, 1968; Thomas, 1971). To see how the elements of the circuit enter the transfer function explicitly, let us label them as in Fig. 4.10. Routine analysis results in the lowpass and bandpass functions

$$T_L(s) = -\frac{1/(R_3R_4C_1C_2)}{s^2 + s/(R_1C_1) + 1/(R_2R_4C_1C_2)} \tag{4.27a}$$

$$T_B(s) = (-sC_2R_4) \times [-T_L(s)] = -\frac{(R_1/R_3) \cdot s/(R_1C_1)}{s^2 + s/(R_1C_1) + 1/(R_2R_4C_1C_2)} \tag{4.27b}$$

The circuit cannot realize the highpass output because we have chosen to merge the summing block with the first integrator. If the highpass output is required, an additional opamp is needed to permit realizing the summing operation (Fig. 2.29a) and the integration (Fig. 3.26a) separately.

Comparing Eq. (4.27a) to the standard form of Eq. (4.17), we identify the appropriate coefficients with the element values as

$$\omega_0^2 = \frac{1}{R_2R_4C_1C_2}, \quad Q = \frac{R_1}{\sqrt{R_2R_4}}\sqrt{\frac{C_1}{C_2}}, \quad \text{and} \quad H = \frac{R_2}{R_3} \tag{4.28}$$

We can now determine the element values to satisfy the given design parameters. This is a typical situation in active filter design: we have more components (here six) than parameters (here three). We will therefore select arbitrarily three of the components and then examine the consequences on the remaining three. Since we used frequency scaling ($\omega_0 = 1$) and intend to use magnitude scaling as well, we make the following choices:

$$C_1 = C_2 = 1 \quad \text{and} \quad R_4 = 1$$

and obtain from Eq. (4.28)

$$R_1 = Q, \quad R_2 = 1, \quad \text{and} \quad R_3 = 1/H$$

These element values give us exactly the circuit previously derived in Fig. 4.9.

An important property of the biquad circuit is that it can be *orthogonally* tuned. By this we mean that

1. R_2 can be adjusted to a specified value of ω_0.

2. R_1 can then be adjusted to give the specified value of Q without changing ω_0, which has already been adjusted.

3. Finally R_3 can be adjusted to give the desired value of H or gain for the circuit, without affecting either ω_0 or Q, which have already been set.

These steps are often called the *tuning algorithm*. This algorithm provides for orthogonal tuning. If this tuning is not possible, then the tuning is called *iterative*, meaning that we try to adjust successively each of the tuning elements until all specifications are met. Orthogonal tuning is always much preferred, especially when the filter is to be produced on a production line with a laser used to adjust each circuit element value.

An example will help us understand the design process:

EXAMPLE 4.2

A lowpass filter is to be designed whose poles in the normalized s-plane are located at $-0.577 \pm j0.8165$. The dc ($\omega = 0$) gain is to be 2. The frequency is scaled by $\omega_0 = 20{,}000$ rad/s ($f_0 = 3183$ Hz). Find the values of the pole frequency and pole quality factor, and design a circuit to realize the specifications. In your design, assume ideal opamps, but test the circuit with LM741 opamps and comment on the results.