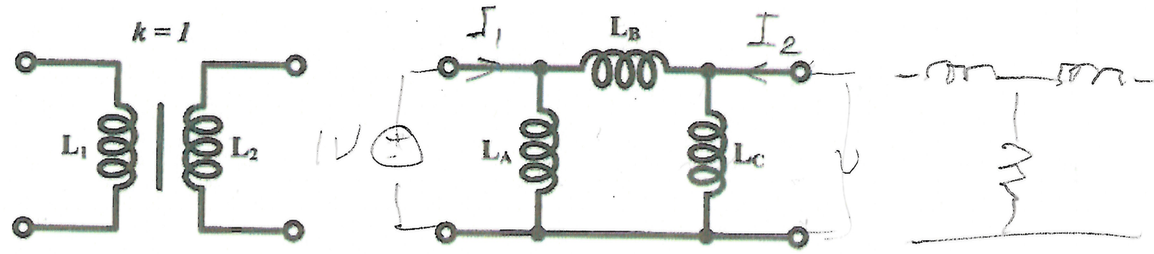
1. Find the values in the "pi"-equivalent shown of the "physical" transformer for general values of k. What happens to the model if $k=1$ (close coupling)?

$M^2 = L_1 L_2$



$$V_1 = sL_1 I_1 + sM I_2$$
$$V_2 = sM I_1 + sL_2 I_2$$

$$\underset{\sim}{Z} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix}$$

$$\underset{\sim}{Y} = \frac{1}{s^2(L_1L_2 - M^2)} \begin{bmatrix} sL_2 & -sM \\ -sM & sL_1 \end{bmatrix}$$

$$y_{11} = \frac{L_2}{s(L_1L_2 - M^2)} = \frac{1}{sL_A} + \frac{1}{sL_B}$$

$$y_{12} = y_{21} = \frac{-M}{s(L_1L_2 - M^2)} = -\frac{1}{sL_B}$$

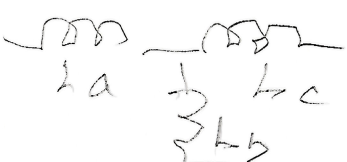
$$y_{22} = \frac{L_1}{s(L_1L_2 - M^2)} = \frac{1}{sL_C} + \frac{1}{sL_B}$$

$$\frac{1}{sL_A} = y_{11} + y_{12} = \frac{L_2 - M}{s(L_1L_2 - M^2)}$$

$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \quad L_C = \frac{L_1L_2 - M^2}{L_1 - M}$$

$$L_B = \frac{L_1L_2 - M^2}{M}, \quad L_1L_2 = M^2/k$$

For $k = 1$, $L_A = L_B = L_C = 0$, the model cannot be used. The T-equivalent is OK:



2.

$$V_{in} = I_{in}\left(\frac{1}{r_{in}} + s(C_{in} + C_{fb})\right)^{-1}$$

$$I_{out} = g_m V_{in} - sC_{fb} V_{in}$$

$$= I_{in}(g_m - sC_{fb})\left[1/r_{in} + s(C_{in} + C_{fb})\right]^{-1}$$

$$A_I = \frac{g_m - j\omega C_{fb}}{1/r_{in} + j\omega(C_{in} + C_{fb})} \qquad \underline{s = j\omega}$$

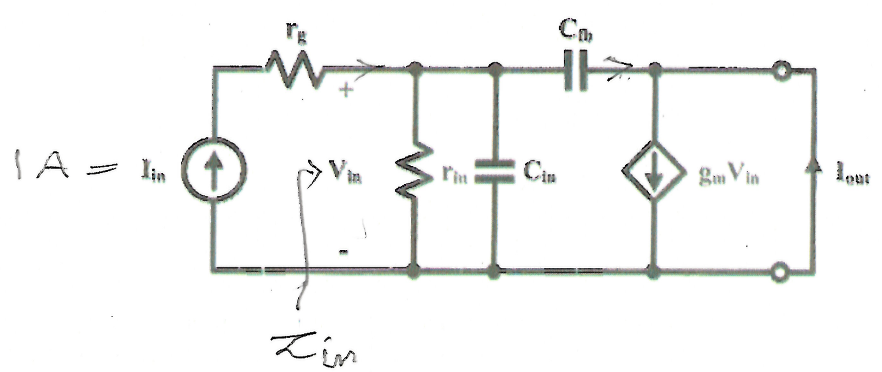
$$|A_I|^2 = \frac{g_m^2 + \omega^2 C_{fb}^2}{1/r_{in}^2 + \omega^2 (C_{in} + C_{fb})^2} \rightarrow 1$$

$$g_m^2 - 1/r_{in}^2 \stackrel{!}{=} \omega_T^2\left[(C_{in} + C_{fb})^2 - C_{fb}^2\right]$$

$$\omega_T^2 = \left[\frac{g_m^2 - 1/r_{in}^2}{C_{in}^2 + 2C_{in}C_{fb}}\right]$$

$$f_T = \frac{1}{2\pi}\left[\frac{g_m^2 - 1/r_{in}^2}{C_{in}^2 + 2C_{in}C_{fb}}\right]^{1/2}$$

2. The circuit shown below is the small-signal model of a semiconductor device. Find
a. the short-circuit current gain $A_I(j\omega) = I_o(j\omega)/I_{in}(j\omega)$, and b. the unit-gain frequency f_T, i.e., the frequency where $|A_I| = 1$.



3. $y_{11}\big|_{s\to 0} \to 1/2sL \doteq 1/Z_1$

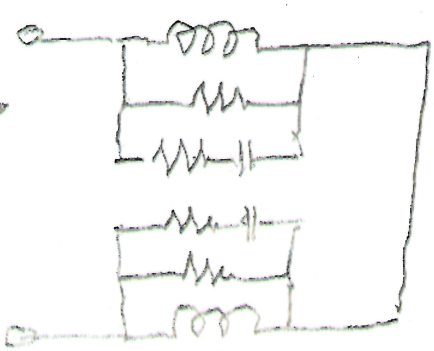
$Z_1 = 2sL$

$Z_{11}\big|_{s\to\infty} \to \frac{1}{2}\left(2R + \frac{1}{sC}\right) = R + \frac{1}{2sC} \doteq R + Z_2$

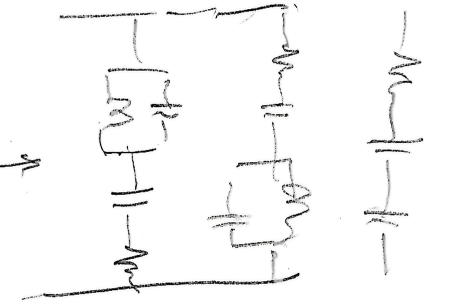
$Z_2 = \frac{1}{2sC}$

Or

$y_{11} \to$



$Z_{11} \to$



$$y_{11} = \frac{1}{2}\left[\frac{1}{sL} + \frac{1}{R} + \frac{1}{R+1/sC}\right] \doteq \frac{1}{Z_1} + \frac{1}{R + RZ_2/(R+Z_2)}$$

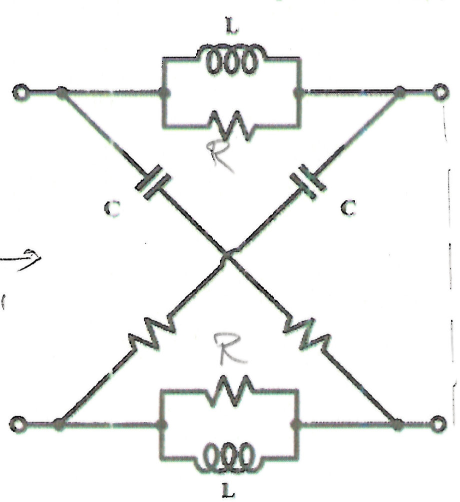
$s \to 0;\ Z_1 = 2sL$

$$\frac{2R + 1/sC}{2R(R+1/sC)} = \frac{2(R+Z_2)}{2(R^2+2RZ_2)} \Rightarrow Z_2 = 1/2sC$$

3. The two two-ports shown below have the same Y, Z, etc. matrices. All resistors have the value R, where $R^2 = L/C$. What are the impedances $Z_1(s)$ and $Z_2(s)$? *Hint:* consider the behavior for very low and very high frequencies!

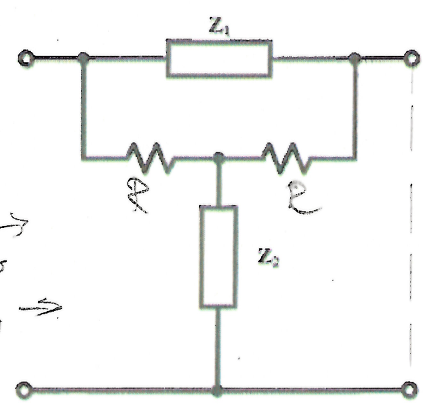
Lattice

$y_{11} \to$



$y_{11} \to$

$Z_{11} \to$



Black box