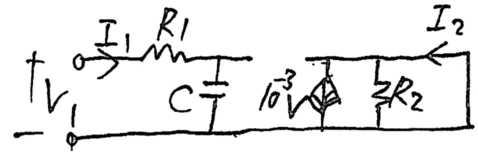
Midterm Solution

1. $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0}$



$$Y_{11} = \frac{1}{R_1+\frac{1}{SC}} = \frac{SC}{SR_1C+1} = \frac{jwC}{jwR_1C+1} = 3\times10^{-4}+2.43\times10^{-4}j$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Since R_2 is shorted, $I_2 = 10^{-3}V$

$$= 10^{-3}.V_1 \cdot \frac{\frac{1}{SC}}{R_1+\frac{1}{SC}}$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = 10^{-3}\frac{\frac{1}{SC}}{R_1+\frac{1}{SC}}$$

$$= 10^{-3}\frac{1}{SR_1C+1}$$

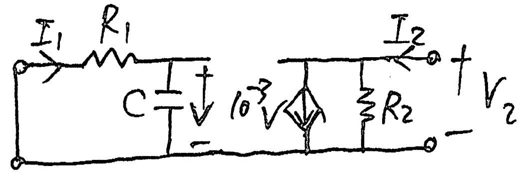
$$= 10^{-3}\cdot\frac{1}{jwR_1C+1}$$

$$= 10^{-3}\frac{1}{j\,2\pi\times10^8\times2\times10^3\times10^{-12}+1}$$

$$= \frac{10^{-3}}{j0.4\pi+1}$$

$$\doteq 3.8\times10^{-4} - 4.8\times10^{-4}j$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$



$\because V_1=0 \quad \therefore V=0$

$\therefore \frac{I_2}{V_2} = \frac{1}{R_2} = \frac{1}{1\times10^{3}} = 10^{-3}$

$\therefore Y_{22} = 10^{-3}$

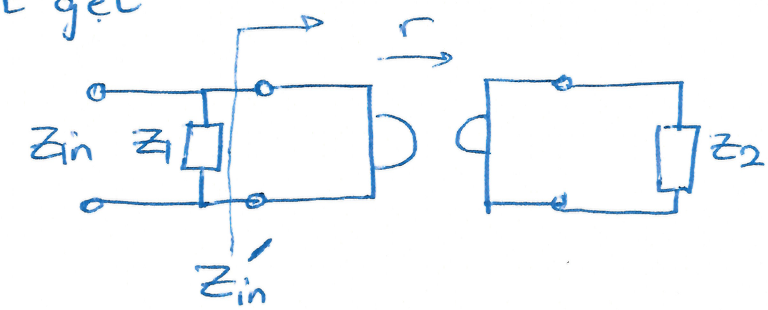
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$\because V_1=0 \quad \therefore I_1=0$

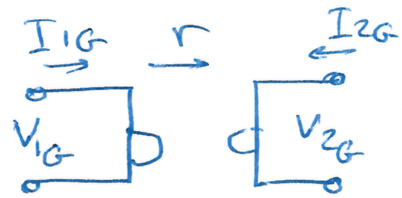
$\therefore Y_{12}=0$

$$\therefore Y = \begin{bmatrix} 3\times10^{-4}+2.43\times10^{-4}j & 0 \\ & \\ 3.8\times10^{-4}-4.8\times10^{-4}j & 10^{-3} \end{bmatrix}$$

2a) We first get



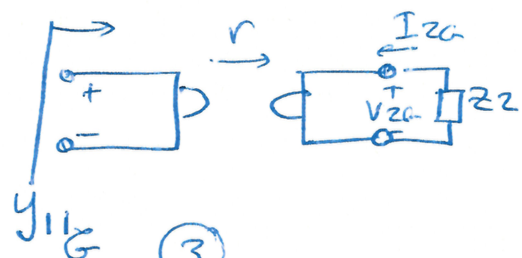
For a gyrator,



Admittance matrix,

$$Y_G = \begin{bmatrix} 0 & 1/r \\ -1/r & 0 \end{bmatrix}$$

Adding output impedance Z_2:



~~y_{11}~~

Solving

$$I_{1_G} = \frac{1}{r} V_{2G} \quad (1)$$

$$I_{2G} = -\frac{1}{r} V_{1_G} \quad (2) \quad \& \quad I_{2G} = -\frac{V_{2G}}{Z_2} = -V_{2G} Y_2 \quad (3)$$

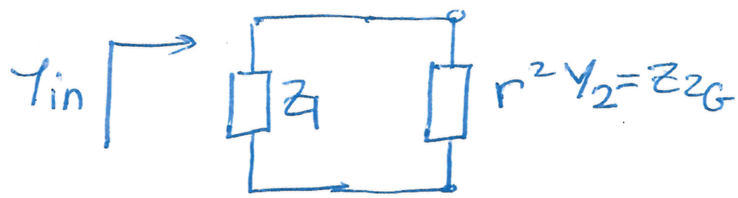
$$\Rightarrow \quad -V_{2G} Y_2 = -\frac{1}{r} V_{1G}$$

$$V_{2G} = \frac{1}{rY_2} V_{1G} \quad (4)$$

$4'$ in (1) : $I_{1G} = \frac{1}{r} \cdot \frac{1}{rY_2} V_{1G}$

$$y_{11_G} = \frac{I_{1G}}{V_{1G}} = \frac{1}{r^2 Y_2}$$

$\therefore$ For complete network



$$Y_{in} = Y_1 + \frac{1}{r^2 Y_2}$$

where $Y_1 = \frac{1}{R} + sC_1$; $\cancel{Z}$ $Y_2 = \frac{1}{R} + sC_2$

$$\boxed{Y_{in} = \frac{1}{R} + sC_1 + \frac{1}{r^2 \cdot \left[\frac{1}{R} + sC_2\right]}}$$

$$\boxed{Z_{in} = Y_{in}^{-1}}$$

b)

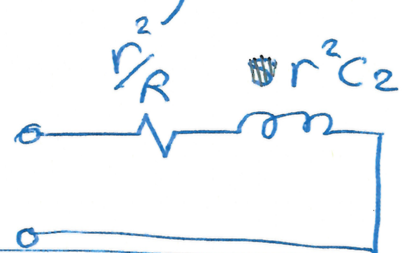
To find passive realization,

Note that

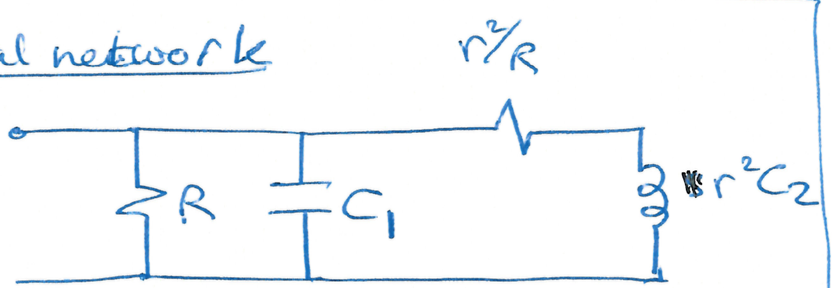
$$Z_2 = r^2 Y_2$$

$$= r^2\left(\frac{1}{R} + sC_2\right)$$

This is equivalent to

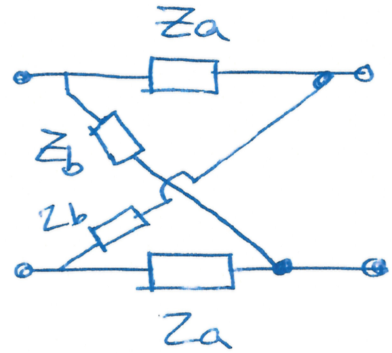


Total network

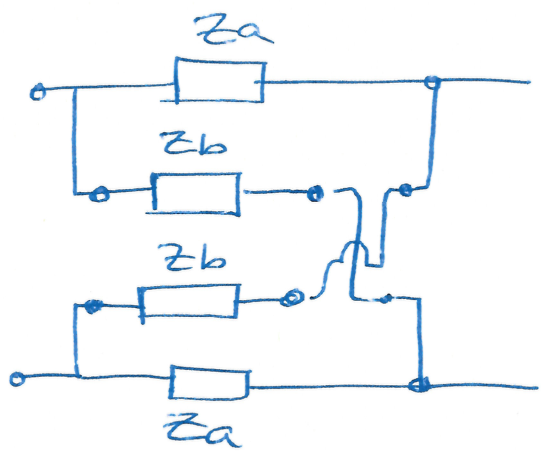


$\boxed{3}$

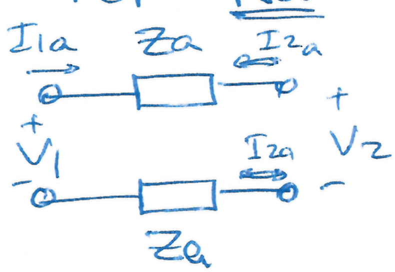
a)



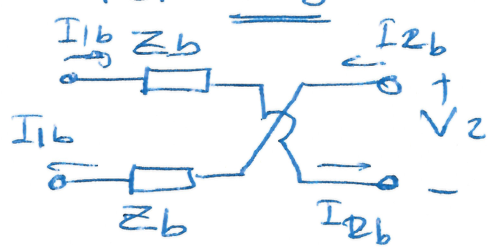
To split the lattice, we can re-draw :



For $\underline{Na}$



For $\underline{N_b}$



Since output are open-circuit, we can only find admittance parameters

$\underline{\underline{Na}}$

$$y_{11_a} = \frac{1}{2Z_a} \; ; \quad y_{21_a} = -\frac{1}{2Z_a}$$

$$y_{22_a} = \frac{1}{2Z_a} \qquad y_{12_a} = -\frac{1}{2Z_a}$$

$\underline{\underline{N_b}}$

$$y_{11_b} = \frac{1}{2Z_b}$$

$$y_{22_b} = \frac{1}{2Z_b}$$

For N_b due to cross-coupling at o/p:

$$y_{12b} = +\frac{1}{2Z_b} \qquad \text{(positive)}$$

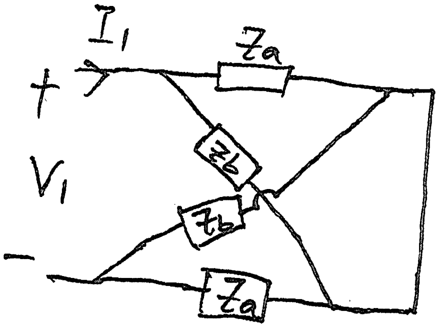
$$y_{21b} = \frac{1}{2Z_b}$$

Since they are in parallel

$$Y_T = Y_a + Y_b = \begin{bmatrix} \frac{1}{2Z_a} + \frac{1}{2Z_b} & \frac{-1}{2Z_a} + \frac{1}{2Z_b} \\ \frac{-1}{2Z_a} + \frac{1}{2Z_b} & \frac{1}{2Z_a} + \frac{1}{2Z_b} \end{bmatrix}$$

3. (b) For a lattice

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$



$$Y_{11} = \frac{1}{(Z_a \| Z_b) + (Z_b \| Z_a)}$$

$$= \frac{1}{\frac{2}{\frac{1}{Z_a}+\frac{1}{Z_b}}}$$

$$= \frac{\frac{1}{Z_a}+\frac{1}{Z_b}}{2}$$

$$= \frac{1}{2Z_a} + \frac{1}{2Z_b}$$

$$= Y_{11a} + Y_{11b}$$

$$\therefore Y = Y_a + Y_b$$