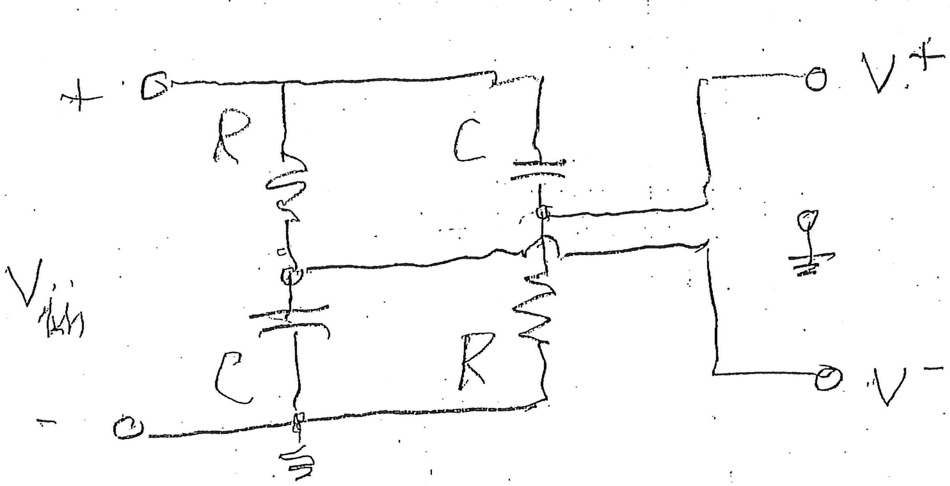
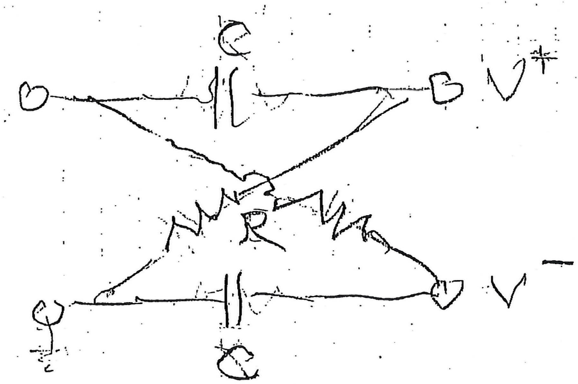
1



$\equiv$



$$\frac{V^+}{V_{in}} = \frac{R}{R+1/sC} = \frac{sT}{1+sT}, \quad \frac{V^-}{V_{in}} = \frac{1}{1+sT} \qquad T=RC$$

$$V^+ - V^- = \frac{-1+j\omega T}{1+j\omega T} V_{in} \longrightarrow \angle H(j\omega) = -2\tan^{-1}(\omega T)$$

$$\angle H \sim -2\omega T, \quad T_{gr} \sim T_{ph} \sim -2T = -2RC$$

$$|H| = 1 \qquad \frac{d\angle H}{d\omega} = -2T$$

2. Gyrator: $\qquad \underset{\sim}{T}$

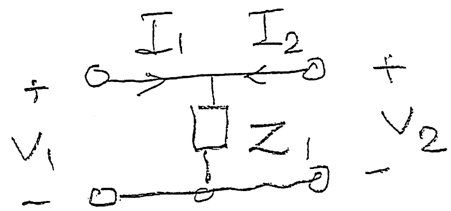
$$V_1 = -R I_2 \qquad V_1 = A V_2 - B I_2$$

$$V_2 = R I_1 \qquad I_1 = C V_2 - D I_2$$

Hence $A = 0$, $B = R$ $\qquad \underset{\sim}{T} = \begin{bmatrix} 0 & R \\ 1/R & 0 \end{bmatrix}$

$C = 1/R$, $D = 0$

Impedance:



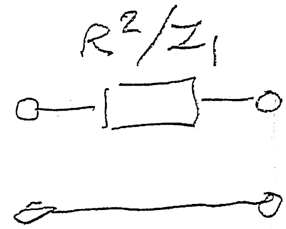
$$V_1 = V_2 = A V_2 - B I_2$$

$$I_1 = V_2/Z_1 - I_2 = C V_2 - D I_2$$

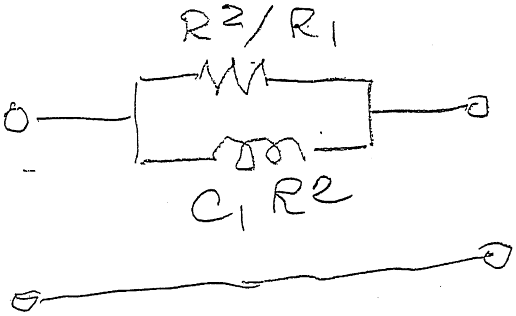
$A_i = 1$, $B_i = 0$

$C_i = 1/Z_1$, $D_i = 1$ $\qquad \underset{\sim}{T}_{imp} = \begin{bmatrix} 1 & 0 \\ 1/Z_1 & 1 \end{bmatrix}$

$$\underset{\sim}{T}_1 = \underset{\sim}{T}\,\underset{\sim}{T}_{imp}\,\underset{\sim}{T} = \begin{bmatrix} 1 & R^2/Z_1 \\ 0 & 1 \end{bmatrix}$$



$$\underset{\sim}{Z}_1 \to \infty, \quad \underset{\sim}{Y}_1 = \frac{Z_1}{R^2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$



3

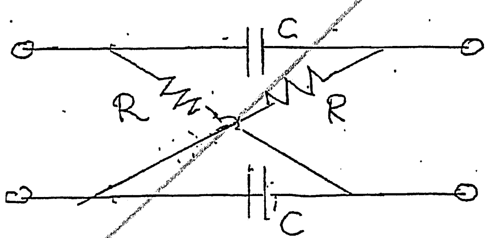
ECE 580

MIDTERM EXAMINATION

October 24, 2018

1. a. Find the voltage ratio V_2/V_1 of the lattice two-port shown.

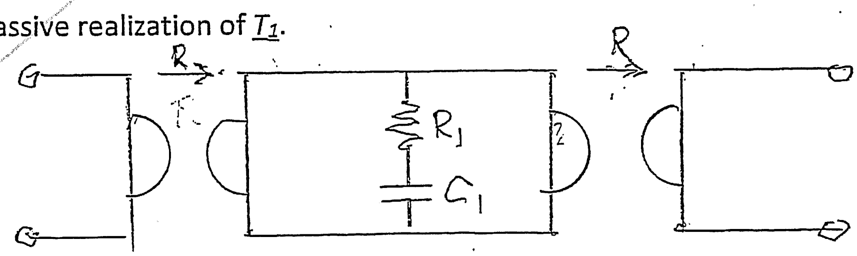
   b. Calculate the gain $|V_2/V_1|$ and the phase shift between V_2 and V_1 as functions of ω.



2. a. Find the chain matrix $\underline{T}$ of a gyrator with gyration resistance R.

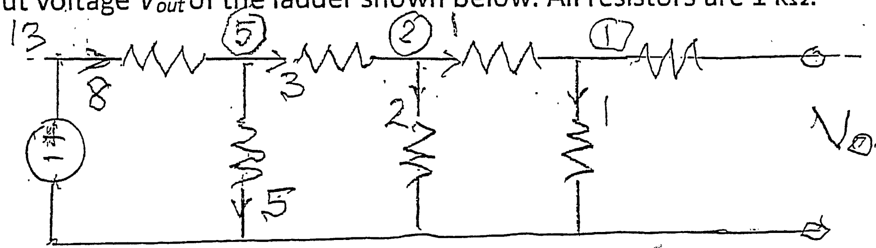
   b. Use your result to find the chain matrix $\underline{T_1}$ of the two-port shown below.

   c. Find a passive realization of $\underline{T_1}$.



3. Find the output voltage V_{out} of the ladder shown below. All resistors are 1 kΩ.

13V ← 1V



Vout → 1V

↓

1/13 V

4. (Extra credit) A two-port T_a has a hybrid matrix $\underline{H_a}$ and another one T_b has a hybrid matrix $\underline{H_b}$.

   a. Show how the matrices should be interconnected so that resulting two-port has a hybrid matrix $\underline{H_a} + \underline{H_b}$.

   b. Derive a test which indicates whether the interconnection gives the correct result.

4 2. We need

$$\underset{\sim}{y} = \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \left[\underset{\sim}{H_a} + \underset{\sim}{H_b} \right] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \underset{\sim}{H}\,\underset{\sim}{x}$$

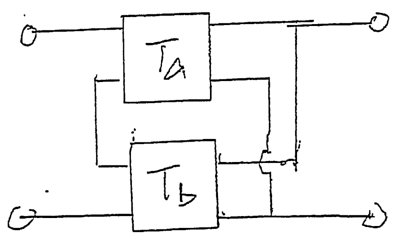
$$\underset{\sim}{y_a} = \underset{\sim}{H_a}\,\underset{\sim}{x_a}$$

$$\underset{\sim}{y_b} = \underset{\sim}{H_b}\,\underset{\sim}{x_b}$$

If $\underset{\sim}{x_a} \equiv \underset{\sim}{x_b} \rightarrow I_{1a} \equiv I_{1b}$ and $V_{2a} \equiv V_{2b}$.

and $\underset{\sim}{y} \equiv \underset{\sim}{y_a} + \underset{\sim}{y_b} \longrightarrow V_1 = V_{1a} + V_b$ and
$I_2 = I_{2a} + I_{2b}$

then $\underset{\sim}{H} = \underset{\sim}{H_a} + \underset{\sim}{H_b}$.

Hence 

Tests

h_{11}, h_{21}