First-Order Filters

Secton 14.4
First-order Switched-capacitor Filter

Active RC Prototype

Switched-capacitor Equivalent

Beginning with conventional continuous-time active RC filters, and replacing each resistor with a switched-capacitor results in a discrete-time filter whose frequency response closely approximates that of the original continuous-time filter at frequencies far below the sampling frequency.
First-order Switched-capacitor Filter

Signal Flow Graph

Switched-capacitor Filter

Once a filter structure is obtained, its precise frequency response is determined through the use of discrete-time analysis.
First-order Switched-capacitor Filter

\[ C_A(1 - z^{-1})V_o(z) = -C_3 V_o(z) - C_2 V_i(z) - C_1(1 - z^{-1})V_i(z) \]

\[ H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{\left(\frac{C_1}{C_A}\right)(1 - z^{-1}) + \left(\frac{C_2}{C_A}\right)}{1 - z^{-1} + \frac{C_3}{C_A}} \]

\[ \left(\frac{C_1 + C_2}{C_A}\right)z - \frac{C_1}{C_A} = -\frac{\left(\frac{C_3}{C_A}\right)z - 1}{\left(1 + \frac{C_3}{C_A}\right)z - 1} \]

\[ z_p = \frac{C_A}{C_A + C_3} \quad z_z = \frac{C_1}{C_1 + C_2} \]

\[ H(1) = \frac{-C_2}{C_3} \]
Frequency response of the first-order filter

\[
H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{\left(\frac{C_1}{C_A}\right)(1 - z^{-1}) + \left(\frac{C_2}{C_A}\right)}{1 - z^{-1} + \frac{C_3}{C_A}} \quad \text{and} \quad -\frac{\left(\frac{C_1}{C_A}\right)(z^{1/2} - z^{-1/2}) + \left(\frac{C_2}{C_A}\right)z^{1/2}}{z^{1/2} - z^{-1/2} + \frac{C_3}{C_A}z^{1/2}}
\]

Using:
\[
z^{1/2} = \cos\left(\frac{\omega T}{2}\right) + j\sin\left(\frac{\omega T}{2}\right) \quad \& \quad z^{-1/2} = \cos\left(\frac{\omega T}{2}\right) - j\sin\left(\frac{\omega T}{2}\right)
\]

\[
H(e^{j\omega T}) \equiv \frac{V_o(e^{j\omega T})}{V_i(e^{j\omega T})} = -\frac{j\frac{2C_1 + C_2}{C_A} \sin\left(\frac{\omega T}{2}\right) + \frac{C_2}{C_A} \cos\left(\frac{\omega T}{2}\right)}{j\left(2 + \frac{C_3}{C_A}\right) \sin\left(\frac{\omega T}{2}\right) + \frac{C_3}{C_A} \cos\left(\frac{\omega T}{2}\right)}
\]

\[
\omega T \ll 1 \Rightarrow \omega \ll 1/T : \quad \left(\frac{\frac{C_1 + C_2/2}{C_A}\omega T + \frac{C_2}{C_A}}{\frac{1 + \frac{C_3}{2C_A}\omega T + \frac{C_3}{C_A}}{j}}\right)
\]

\[
j\omega T = -\frac{C_2}{C_1} \quad \text{and} \quad j\omega T = -\frac{C_3}{C_A}
\]

\[
1 + \frac{C_2}{2C_1} \quad \text{and} \quad 1 + \frac{C_3}{2C_A}
\]
Switch Sharing

Fig. 14.18(a)  Fig. 14.19
Fully Differential Implementation

Allows for signal inversion by crossing wires
Biquad Filters

Section 14.5
Discrete-Time Biquad Realizations

• Similar to continuous-time biquads, discrete-time biquads may be realized using two integrators in a negative feedback loop, one of the integrators having loss to provide stability.

• Several different discrete-time biquads can be realized by varying the type (i.e., delayed, delay-free) and exact connection of switched-capacitor branches within the two-integrator loop.

• The choice of which biquad to use depends upon the specific pole and zero locations sought.
Biquad #1

Active RC Prototype

Swapped-capacitor Equivalent
Biquad #1

Signal Flow Graph

Switched-capacitor Equivalent
Biquad #1 Transfer Function

\[ H(z) \equiv \frac{V_o(z)}{V_i(z)} = \frac{-(K_2 + K_3)z^2 + (K_1K_5 - K_2 - 2K_3)z + K_3}{(1 + K_6)z^2 + (K_4K_5 - K_6 - 2)z + 1} \]
Biquad #1 Design Equations

\[ H(z) = \frac{-a_2z^2 + a_1z + a_0}{b_2z^2 + b_1z + 1} = \frac{(K_2 + K_3)z^2 + (K_1K_5 - K_2 - 2K_3)z + K_3}{(1 + K_6)z^2 + (K_4K_5 - K_6 - 2)z + 1} \]

\[ K_3 = a_0 \]

\[ K_2 = a_2 - a_0 \]

\[ K_1K_5 = a_0 + a_1 + a_2 \]

\[ K_6 = b_2 - 1 \]

\[ K_4K_5 = b_1 + b_2 + 1 \]

Extra degree of freedom in choice of \( K_5 \) determines the amplitude of the signal at \( V_1 \).
Biquad #2

Active RC Prototype

Switched-capacitor Equivalent
Biquad #2 Design Equations

\[ H(z) = -\frac{a_2 z^2 + a_1 z + a_0}{z^2 + b_1 z + b_0} = -\frac{K_3 z^2 + (K_1 K_5 + K_2 K_5 - 2 K_3) z + (K_3 - K_2 K_5)}{z^2 + (K_4 K_5 + K_5 K_6 - 2) z + (1 - K_5 K_6)} \]

\[ K_1 K_5 = a_0 + a_1 + a_2 \]
\[ K_2 K_5 = a_2 - a_0 \]
\[ K_3 = a_2 \]
\[ K_4 K_5 = 1 + b_0 + b_1 \]
\[ K_5 K_6 = 1 - b_0 \]

Extra degree of freedom in choice of \( K_5 \) determines the amplitude of the signal at \( V_1 \).