Technical Report

ON CONVERGENCE PROOFS FOR PERCEPTRONS

Prepared for:

OFFICE OF NAVAL RESEARCH
WASHINGTON, D.C.

CONTRACT Nonr 3438(00)

By: *Albert B. J. Novikoff*

STANFORD RESEARCH INSTITUTE

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ABSTRACT

A short proof is given of the convergence (in a finite number of steps) of an algorithm for adjusting weights in a single-threshold device. The algorithm in question can be interpreted as the error-correction procedure introduced by Rosenblatt for his "α-Perceptron." The proof presented extends the basic idea to continuous as well as discrete cases, and is interpreted geometrically.

CONTENTS

I INTRODUCTION

The purpose of this report is to exhibit an extremely short and, more notably, transparent proof of a theorem concerning perceptrons. The theorem itself must now be considered one of the most basic theorems about perceptrons, and indeed, is among the first theorems proved by Rosenblatt and his collaborators. It also enjoys the peculiar distinction of being one of the most often re-proved results in the field. The succession of proofs now available progresses from somewhat cloudy statements (which at one time caused doubt among "reasonable men" that the theorem was true) to comparatively crisp statements of a purely mathematical nature which nonetheless use more print than is strictly necessary.[1-6]

More to the point, latter-day proofs fail to enunciate a simple principle involved. This principle permits one to modify the hypotheses in a variety of ways and secure similar results; it may well be useful in establishing genuinely new theorems of like character. We therefore present our proof in its entirety, in part to verify our claim that it is as short a line as can be drawn from hypotheses to conclusion, and also with the hope of terminating an already lengthy process of successive refinements. In addition, we prove a related theorem which is the continuous analogue of the perceptron theorem, and we indicate that various other theorems may be obtained by appropriately modifying the hypotheses. We also discuss a geometrical interpretation of the perceptron theorem in terms of a convex cone and its dual.

II STATEMENT OF THE THEOREM

Whereas previous proofs of the theorem appealed to a structure, called by Rosenblatt and his co-workers an α-perceptron, the present theorem, proof, and discussion apply without modification to a structure consisting of a single threshold element acting on a weighted set of inputs.

Theorem: Let $w_1, \ldots, w_N$ be a set of vectors in a Euclidean space of fixed finite dimension, satisfying the hypothesis that there exists a vector y such that

$$(w_i, y) > \theta > 0 \qquad i = 1, \ldots, N \quad . \tag{1}$$

Consider the infinite sequence $w_{i_1}, w_{i_2}, w_{i_3}, \ldots,\ 1 \leq i_k \leq N$ for every k, such that each vector $w_1, \ldots, w_N$ occurs infinitely often. Recursively construct a sequence of vectors $v_0, v_1, \ldots, v_n, \ldots$ as follows:

v_0 is arbitrary

$$v_n = \begin{cases} v_{n-1} & \text{if } (w_{i_n}, v_{n-1}) > \theta \\ v_{n-1} + w_{i_n} & \text{if } (w_{i_n}, v_{n-1}) \leq \theta \end{cases} \quad . \tag{2}$$

The sequence $\{v_n\}$ is convergent—*i.e.*, for some index m, $v_m = v_{m+1} = v_{m+2} = \ldots = \tilde{v}$.

Remarks:

(1) In particular, the theorem insures that $(w_i, \tilde{v}) > \theta$ for $i = 1, \ldots, N$, since each w_i occurs arbitrarily far out in the sequence $\{w_{i_k}\}$. It is only to obtain this consequence that we impose the restriction that each w_i occurs infinitely often in the training sequence.

(2) Theoretically, we may take $\theta = 0$ without loss of generality.[1] However, this often has the effect of smuggling in numbers of large magnitude. For this reason, we retain the general θ, but in the concluding section we do consider the relation between the general case and the case $\theta = 0$.

(3) In the private language of perceptron workers, the theorem reads as follows: A set of incoming signals is divided into two adjacent classes. A "satisfactory" assignment of weights from the associator units is defined as an assignment resulting in a response +1 for signals of Class I, and −1 for signals of Class II. The theorem asserts that no matter what assignment of weights we begin with, the process of recursively readjusting the weights by the method known as "error correction" will terminate after a finite number of corrections in a satisfactory assignment, provided such a satisfactory assignment exists. More briefly, a finite number of corrections will teach the perceptron to perform any given dichotomy of signals, if the dichotomy is within the capacity of the perceptron at all.

The definition of α-perceptron and the precise correspondence between the theorem's original verbal description and the purely mathematical assertion of the above theorem are provided by Block.[1] A brief glossary indicating the correspondence follows: the vector w_i represents the activity of the associators, including class information, when stimulus S_i is presented. The vector y, which we assume to exist, represents a "satisfactory" assignment of associator weights; y has as many components as there are associators. The sequence $\{w_{i_n}\}$ represents the "training sequence," and the rule for defining $\{v_n\}$ describes the error-correction procedure. The positive number θ is a threshold which must be exceeded for the response of the perceptron to be correct; a vector v such that $(w_i, v) > \theta$ is an assignment of associator outputs which successfully classifies the ith signal. [If $(w_i, v) \leq \theta$, then either the ith signal has been classified as belonging to the incorrect class or the perceptron has refrained from commitment, depending on whether or not the inequality is strict.]

(4) It is clear that because of Eq. (2), as n varies, the sequence v_n changes, if at all, only by the addition of one or another of the set $w_1, \ldots, w_N$. For this reason "convergence" implies "convergence in a finite number of steps." The word "stabilizes" has been suggested to describe this kind of convergence.

III PROOF OF THEOREM

We may omit from the training sequence all terms w_{i_n} for which $v_n = v_{n-1}$, as these w_{i_n} are clearly inessential. The new training sequence is such that correction takes place at every step. Adjusting our notation, we may assume that

$$v_n = v_{n-1} + w_{i_n} \quad \text{and} \quad (w_{i_n}, v_{n-1}) \leq \theta \quad \text{for each } n \quad . \tag{3}$$

We observe that n is the number of corrections made up to the nth step. The assertion of the theorem after this change of notation is that n can range only through a finite set of integers—that is, conditions (1) and (3) cannot continue to hold simultaneously for all $n = 1,2,3,\ldots$.

First we show that inequality (1) alone implies the inequality

$$\|v_n\|^2 > Cn^2 \tag{4}$$

for suitable choice of the positive constant C, and n sufficiently large. Since $v_n = v_0 + w_{i_1} + \ldots + w_{i_n}$, inequality (1) implies that v_n satisfies $(v_n, y) > (v_0, y) + n\theta$. Using the Cauchy-Schwartz inequality,

$$\|v_n\|^2 \geq \frac{(v_n, y)^2}{\|y\|^2} > \frac{[(v_0, y) + n\theta]^2}{\|y\|^2} = \frac{\theta^2}{\|y\|^2}\left[n + \frac{(v_0, y)}{\theta}\right]^2$$

If $(v_0, y) \geq 0$, we may choose $C = \theta^2/\|y\|^2$, and inequality (4) is satisfied for all n. If $(v_0, y) < 0$, we may choose $C = (1/4)(\theta^2/\|y\|^2)$, and inequality (4) is satisfied for $n > -2[(v_0, y)/\theta]$.

On the other hand, we show that inequalities (3) alone imply the inequality

$$\|v_n\|^2 \leq \|v_0\|^2 + (2\theta + M)n \tag{5}$$

where

$$M = \max_{i=1,\ldots,N} \|w_i\|^2 \quad .$$

Using inequality (3), the integer-argument function $\|v_k\|^2$ satisfies for each k the difference inequality

$$\|v_k\|^2 - \|v_{k-1}\|^2 = 2(v_{k-1}, w_{i_k}) + \|w_{i_k}\|^2 \leq 2\theta + M \quad .$$

Adding the inequalities for $k = 1,2,\ldots,n$, we obtain inequality (5).

Clearly, inequalities (4) and (5) are incompatible for n sufficiently large.

IV AN ANALOGOUS THEOREM

The theorem of Section II is the discrete analogue of the following theorem, which may seem more intuitive: Let $v(t)$ be a curve in Euclidean n-space described by a smooth vector function of the continuous variable t, such that there exists a vector y such that

$$\left(\frac{dv}{dt}\ ,\ y\right) > C > 0 \tag{1'}$$

and

$$\frac{1}{2}\frac{d}{dt}\ \|v(t)\|^2 = \left[\frac{dv}{dt}\ ,\ v(t)\right] \leq \theta\ , \quad 0 \leq t < b \quad . \tag{3'}$$

There exists an upper bound for b; in particular, inequalities (1)′ and (3)′ are compatible only over a finite domain on the t-axis.

The proof is virtually identical with that of the discrete case. Integrating inequality (1)′ from 0 to t, we obtain

$$[v(t),y] > [v(0),y] + Ct \quad . \tag{6}$$

Integrating inequality (3)′ from 0 to t, we obtain

$$\|v(t)\|^2 \leq 2\theta t + \|v(0)\|^2 \quad . \tag{7}$$

Using the Cauchy-Schwartz inequality and inequality (6), we obtain

$$\|v(t)\|^2 \geq \frac{[v(t),y]^2}{\|y\|^2} > \frac{\{[v(0),y] + Ct\}^2}{\|y\|^2} \quad . \tag{8}$$

Inequalities (7) and (8) together show that t cannot exceed the larger root of the quadratic

$$\{[v(0),y] + Ct\}^2 = \|y\|^2\{2\theta t + \|v(0)\|\}^2 \quad .$$

Remarks:

(1) Inequality (1)′ means that the tangent vector to the curve lies on one side of a hyperplane. Inequality (3)′ means that the rate of growth of $\|v\|^2$ is bounded.

(2) We may compare the above argument for the continuous case with the extremely familiar phenomenon that

$$\left[v(t)\ ,\ \frac{dv}{dt}\right] = 0 \tag{9}$$

implies

$$\|v(t)\|^2 \quad \text{is constant} \quad ; \tag{10}$$

i.e., a curve whose tangent vector is always perpendicular to its position vector is constrained to lie on a sphere. Replacing the orthogonality condition (9) with the inequality (3)′ results in an inequality (7) on the rate of growth of the function $f(t) = \|v(t)\|^2$, which is clearly a weakening of the condition (10) that $f(t)$ be constant.

(3) The principle involved in the theorem is the following: The condition of (3)′, that the tangent vector have bounded scalar product with the position vector, clearly results in an upper bound for the instantaneous position of the curve as a function of time. On the other hand, if the tangent vectors dv/dt to the curve remain sufficiently large and do not depart too badly from colinearity, as prescribed, for example, by (1)′, then a lower bound on the cumulative growth results, as in (8). This is intuitively clear: If dv/dt does not get too small, the total arclength will increase with at least a certain rate. If, on the other hand, the dv/dt are sufficiently "nearly colinear" then the serpentine path swept out by $v(t)$ cannot reverse its direction enough to prevent its over-all migration away from its starting point. The opposition of these two influences implies the termination of one of the two relations (1)′ or (3)′.

We will not dwell upon the matter of how assorted variations of this theme will continue to produce assertions that t, or, in the discrete case, n, must remain bounded. Whether each of these deserves to be dignified with the name *theorem* is a moot point.

V GEOMETRICAL INTERPRETATION

We conclude with a few words about the geometrical interpretation of the assumption (1). We assume familiarity with the theory of convex sets in Euclidean vector spaces. For the most part, we state these remarks without proof. For a general introduction to this theory, see Blackwell and Girshick,[9] and Gale.[10]

The *polyhedral cone*, C, with generators, $w_1,\ldots,w_N$, is defined as all vectors of the form $\lambda_1 w_1 + \ldots + \lambda_N w_N$, where $\lambda_i \geq 0$, $i = 1,2,\ldots,N$. The cone C is called *proper* if for all $v \neq 0$, C never contains both v and $-v$; or equivalently, if C, apart from its vertex, lies in the *interior* of a half space. The condition that there exists a vector y satisfying (1) is precisely equivalent to the condition that C is a proper cone.

We remark that requiring the existence of a vector y which satisfies (1) with $\theta > 0$, is neither stronger nor weaker than requiring the existence of a vector $\tilde{y}$ which satisfies (1) with $\theta = 0$—*i.e.*, which satisfies

$$(w_i,\tilde{y}) > 0 \ , \qquad i = 1,\ldots,N \ . \qquad (1)''$$

Indeed, y itself can serve for $\tilde{y}$; conversely, given θ and $\tilde{y}$ satisfying (1)″, any sufficiently large positive multiple $y = \lambda\tilde{y}$ of $\tilde{y}$ with

$$\lambda > \frac{\theta}{\min\limits_{i=1,\ldots,N} (w_i,\tilde{y})} \ ,$$

will satisfy (1). Condition (1)″ is the customary way of specifying that the cone C be proper.

For the continuous analogue, requiring that infinitely many vectors dv/dt satisfy (1)′ with $c > 0$ is actually stronger than requiring only that dv/dt satisfy

$$\left(\frac{dv}{dt} \ , \ y\right) > 0 \ , \qquad 0 \leq t < b \ . \qquad (1)'''$$

In fact, the left-hand side, though positive for each t, need not be bounded away from zero. If we assume $(1)'''$ to hold for $0 \leq t \leq b$ (equality permitted at b) and dv/dt to be continuous, then, as in the discrete case, it is true that $(1)'''$ and $(1)'$ are precisely equivalent.

The cone C^* of all vectors v such that $(w,v) \geq 0$ for all w in C, or equivalently such that

$$(w_i,v) \geq 0 , \quad i = 1,\ldots,N \quad , \tag{10}$$

where C is the polyhedral cone generated by $w_1,\ldots,w_n$, is called the *dual cone* of C. Its interior consists of all v for which every inequality in (10) is strict. The bigger C is the smaller C^* is, and vice versa; for example, when C is a half-space, C^* is a half-line. In general, for $n > 2$ neither need include the other. On the other hand, it is not possible to weakly separate C and C^*—*i.e.*, there is no $z \neq 0$ such that both $(w,z) \geq 0$ for all w in C, and $(v,z) \leq 0$ for all v in C^*. If such a z did exist, then by the first inequality, z is in C^*; then choosing v to be z in the second inequality implies that $(z,z) \leq 0$—*i.e.*, that $z = 0$, contrary to the assumption that $z \neq 0$.

When C is proper, C^* has an interior; indeed, the $\tilde{y}$ of $(1)''$ is in the interior of C^*. If C^* has an interior, C and the interior of C^* overlap; if not, then a consequence would be that C and C^* could be weakly separated by a classic result on the separation of convex sets. This we have seen to be impossible.

As previously observed, if y is in the interior of C^*, then a suitable positive multiple of y will satisfy (1). Let the set of vectors satisfying (1) be denoted by D. D is a subset of the interior of C^*. The relation between (1) and the dual cone defined by (10) may be summarized as follows: (1) has a solution (*i.e.*, D is non-empty) if and only if C^* has an interior.

The error-correction procedure is a *recursive construction* of a vector in D of the form

$$k_1 w_1 + \ldots + k_N w_N$$

where k_i is the number of times w_i occurred in the irredundant training sequence before the termination of the correction process. The mere existence of such a vector (without a construction algorithm) is assured by the fact that C and D overlap, which is a consequence of the above-mentioned overlap between C and the interior of C^*.

In general, if condition (1) is fulfilled, so that $w_1, \ldots, w_N$ generate a proper cone, it will happen that some subset of the w's will generate the same cone (which then has the same dual C^*), and it suffices to restrict attention to this subset in constructing the training sequence. To accelerate the termination of the correction process one should use for correction those w's which themselves are nearest the interior of C^*, and which are as long as possible. If, for example, $w_3 = w_1 + w_2$, the single addition of w_3 will accomplish as much as the successive additions of w_1 and w_2. The question of whether a dichotomy is within the combinatorial capacity of an α-type perceptron reduces to whether or not C^* has an interior, or equivalently, whether or not C is proper. This question is discussed in a somewhat different context by Joseph and Hay,[7] and Keller.[8]

After the work was completed it was pointed out that S. Agmon had previously considered[11] a variety of similar correction procedures (none, however, identical with the above) and showed their convergence in general to be at a geometric rate.

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