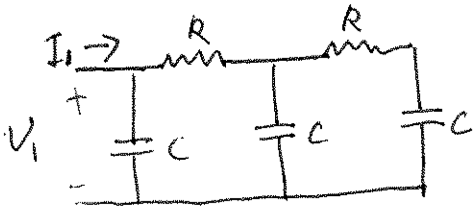
Solution:



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \frac{1}{sC} \,\|\, \left[R + \frac{1}{sC} \,\|\, \left(R + \frac{1}{sC}\right)\right]$$

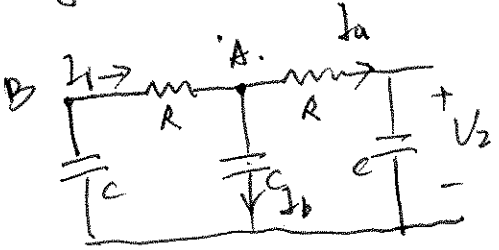
$$= \frac{1}{sC} \,\|\, \frac{R + \frac{1}{sC} + R^2 sC + 2R}{RsC + 2}$$

$$= \frac{3R + \frac{1}{sC} + R^2 sC}{RsC + 2} \cdot \frac{1}{sC} \Big/ \frac{3R + \frac{1}{sC} + R^2 Cs}{RCs + 2} + \frac{1}{sC}$$

$$= 3R + \frac{1}{sC} + R^2 C s \Big/ 3RCs + 1 + R^2C^2s^2 + RCs + 2.$$

$$= \frac{R^2C^2s^2 + 3RCs + 1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs} = \frac{1e{-}18s^2 + 3e{-}9s + 1}{1e{-}30s^3 + 4e{-}21s^2 + 3e{-}12s}$$

symmetrical network $\Rightarrow Z_{11} = Z_{22}$



$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

~~Apply~~ KCL KVL equation, we have

$$V_A = \left(R + \frac{1}{sC}\right) \cdot I_a = RCs + 1$$

$$I_b = V_A / \tfrac{1}{sC} = RC^2s^2 + Cs$$

$$V_B = (I_a + I_b) \cdot R + V_A = R^2C^2s^2 + 3RCs + 1$$

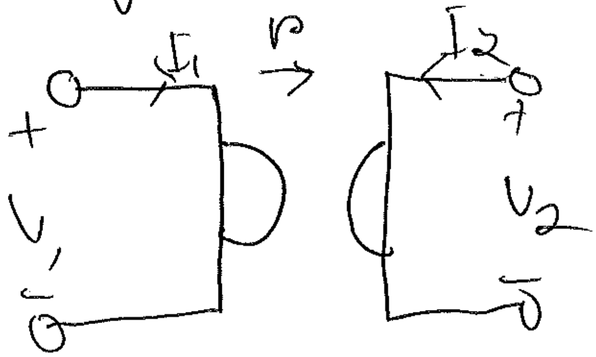
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = 1 \Big/ R^2C^3s^3 + 4RC^2s^2 + 3Cs = 1 \Big/ 1e{-}30s^3 + 4e{-}21s^2 + 3e{-}12s$$

$Z_{12} = Z_{21} \Leftarrow$ reciprocal network.

$$Z = \begin{bmatrix} \frac{R^2C^2s^2 + 3RCs + 1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs} & \frac{1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs} \\ \frac{1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs} & \frac{R^2C^2s^2 + 3RCs + 1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs} \end{bmatrix} = \begin{bmatrix} \frac{1e{-}18s^2 + 3e{-}9s + 1}{1e{-}30s^3 + 4e{-}21s^2 + 3e{-}12s}, & \frac{1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs}; \\ \frac{1}{R^2C^3s^3 + 4RC^2s^2 + 3Cs}, & \frac{1e{-}18s^2 + 3e{-}9s + 1}{1e{-}30s^3 + 4e{-}21s^2 + 3e{-}12s} \end{bmatrix}$$

Question 2

a) Gyrator



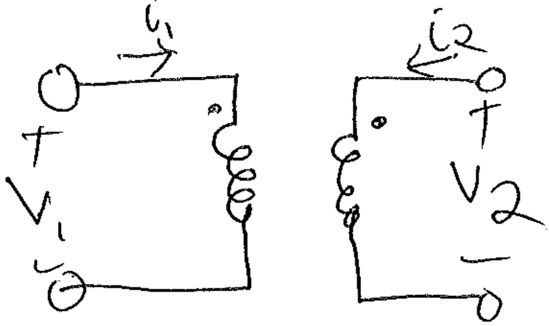
$v_1 = -r i_2$

$v_2 = r i_1$

* Many possible but two simplest

$$\underbrace{\underline{\underline{Z}} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}}_{\text{in notes}} \qquad \underline{\underline{Y}} = \begin{bmatrix} 0 & 1/r \\ -\frac{1}{r} & 0 \end{bmatrix}$$

b) Ideal Transformer (as defined in notes)



$V_1 = n V_2$

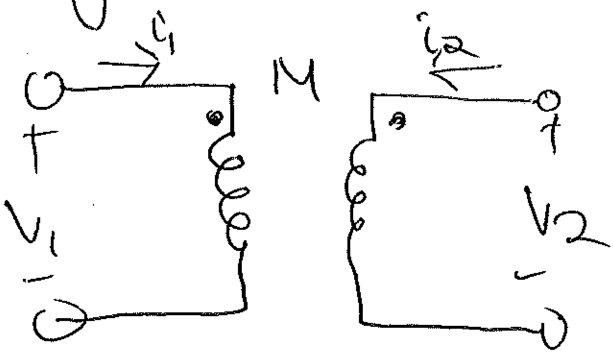
$i_2 = -n i_1$

$$\underline{\underline{H}} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix}$$

* in notes

c)

c) Physical Transformer



$$V_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = M\frac{di_1}{dt} + \frac{L_2 di_2}{dt}$$

in laplace domain

$$V_1 = sL_1 i_1 + sMi_2$$

$$V_2 = sL_2 i_2 + sMi_1$$

$$\underline{\underline{Z}} = \begin{bmatrix} sL_1 & sM \\ sM & sL_2 \end{bmatrix}$$