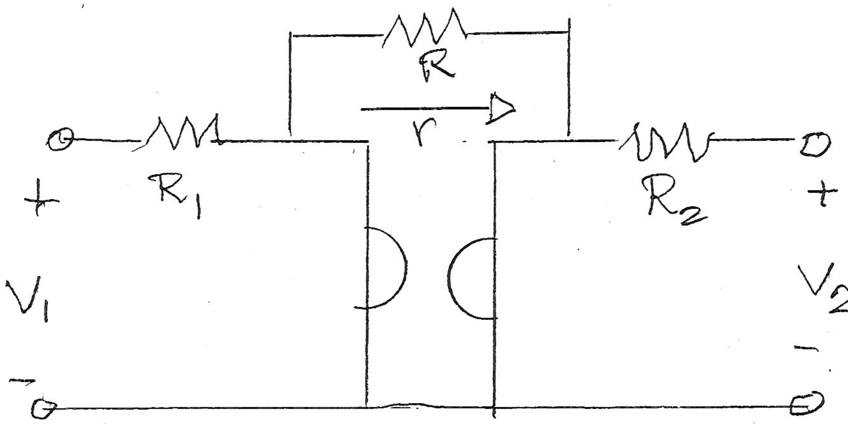
ECE 580

HOMEWORK 3

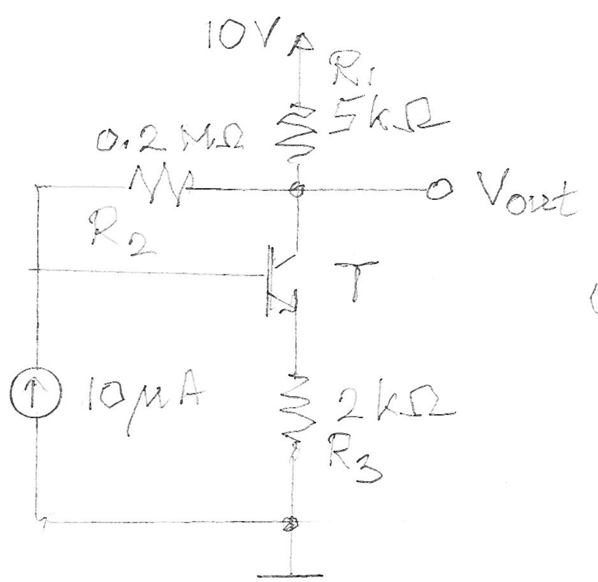
Due Nov. 11, 2015

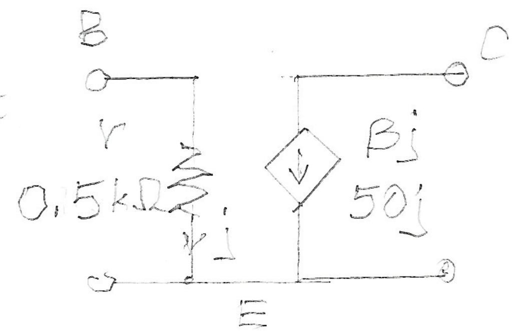
1. a. Find the scattering matrix of the circuit shown. Assume that all resistors (including the gyration resistance R) equal 50 Ω.

   b. What can you state about the physical properties of the two-port?



2. Find the sensitivities of the output voltage of the amplifier shown to variations of all circuit parameters (including β).





T model

Problem 3: (*10 points*) For the two-port of Problem 2, assume r = R = 1 Ω. Choose as its terminations R1 = R2 = 1Ω. Find the scattering matrix, and comment on the transmission properties of the circuit.

For $r = R = 1\Omega = R_1 = R_2$,

$$\underset{\sim}{Z} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad \underset{\sim}{Z_a} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} = \underset{\sim}{Z_{an}}$$

$$\underset{\sim}{Y_{an}} = \frac{1}{4} \begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix}$$

In our case, Z parameter is different by 50 times but S parameter is the same since they are normalized by termination impedance

$$\underset{\sim}{S} = \underset{\sim}{1} - 2\underset{\sim}{Y_{an}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$S_{11} = S_{22} = 0$: matching at both ports at all frequencies; max. power entering.

$S_{12} = 0$: if excited at port 2, no power at port 1 → all power steered to R.

$S_{21} = 1$: if excited at port 1, max. power transfer into R_2, no power in R ($V_2 = V_1$).

3. Analyze the sensitivities of the small-signal output v_O to variations of R_1, R_2, R_3, r and β using the given transistor model. What is the possible range of v_O if all tolerances are 5 percent?

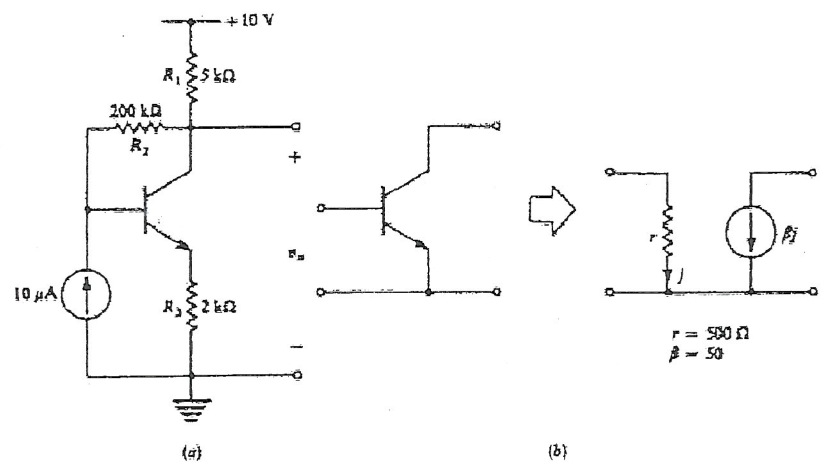


Figure 9-19 (a) Transistor circuit; (b) transistor model.

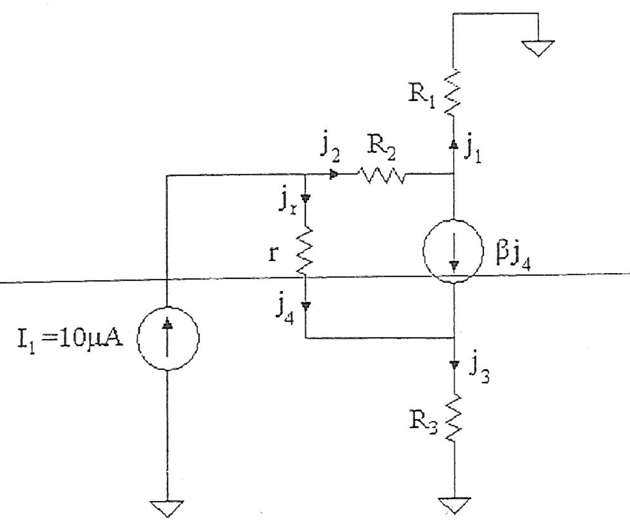


Figure 9-19c Physical network with small-signal model

From the physical network

$I_1 = 10\mu A = j_2 + j_r$

$j_2 = I_1 - j_r$

$j_3 = j_r + \beta j_r = j_r(\beta + 1)$

$j_1 = j_2 - \beta j_r$

$j_1 * R_1 + j_2 * R_2 - j_r * r - j_3 * R_3 = 0$ (loop through all resistors)

$(j_2 - \beta j_r) * R_1 + j_2 * R_2 - j_r * r - j_r(\beta + 1) * R_3 = 0$

$(I_1 - j_r - \beta j_r) * R_1 + (I_1 - j_r) * R_2 - j_r * r - j_r(\beta + 1) * R_3 = 0$

$(I_1 - j_r(\beta + 1)) * R_1 + (I_1 - j_r) * R_2 - j_r * r - j_r(\beta + 1) * R_3 = 0$

$(10\mu A - j_r(50 + 1)) * (5k\Omega) + (10\mu A - j_r) * (200k\Omega) - j_r * (500\Omega) - j_r(50 + 1) * (2k\Omega) = 0$

$0.05V - j_r(255k\Omega) + 2V - j_r(200k\Omega) - j_r * (500\Omega) - j_r * (102k\Omega) = 0$

$j_r(557.5k\Omega) = 2.05V$

$j_r = 3.6771\mu A$

$j_2 = I_1 - j_r = 10\mu A - 3.6771\mu A = 6.3228\mu A$

$j_3 = j_r(\beta + 1) = 3.6771\mu A * 51 = 187.534\mu A$

$j_1 = j_2 - \beta j_r = 6.3228\mu A - 50 * 3.6771\mu A = -177.534\mu A$

$j_4 = j_r = 3.6771\mu A$

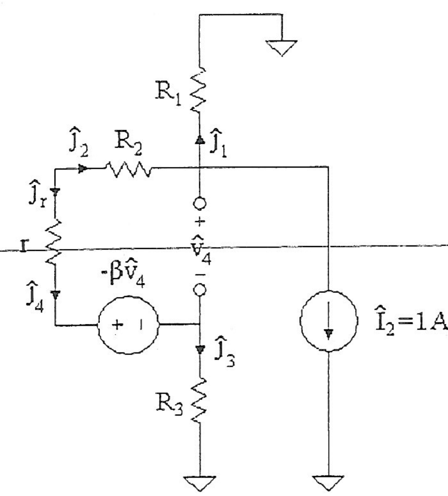


Figure 9-19d

From the adjoint network

$\hat{I}_2 = 1A = \hat{j}_2 - \hat{j}_1$

$\hat{j}_1 = \hat{j}_2 + \hat{I}_2$

$\hat{j}_2 = -\hat{j}_r = -\hat{j}_4$

$\hat{j}_1 * R_1 + \hat{j}_2 * R_2 - \hat{j}_r * r + \beta \hat{v}_4 - \hat{j}_3 * R_3 = 0$ (loop 1)

$$\left(\hat{j}_2 + \hat{I}_2\right) * R_1 + \hat{j}_2 * R_2 + \hat{j}_2 * r + \beta\hat{v}_4 + \hat{j}_2 * R_3 = 0$$

$$\hat{j}_2(R_1 + R_2 + R_3 + r) + \beta\hat{v}_4 = -\hat{I}_2 * R_1 \qquad (1)$$

$$\hat{j}_1 * R_1 - \hat{v}_4 + \hat{j}_3 * R_3 = 0$$

$$\left(\hat{j}_2 + \hat{I}_2\right) * R_1 - \hat{v}_4 + \hat{j}_2 * R_3 = 0$$

$$\hat{j}_2(R_1 + R_3) - \hat{v}_4 = -\hat{I}_2 * R_1$$

$$\hat{j}_2\beta(R_1 + R_3) - \beta\hat{v}_4 = -\hat{I}_2 * \beta R_1 \qquad (2)$$

Adding (1) and (2) we get

$$\hat{j}_2(R_1(\beta + 1) + R_2 + R_3(\beta + 1) +)r = -\hat{I}_2 * R_1(\beta + 1)$$

$$\hat{j}_2(557.5k\Omega) = 255kV$$

$$\hat{j}_2 = 0.45740A$$

$$\hat{j}_1 = \hat{j}_2 + \hat{I}_2 = 0.45740A - 1A = -0.54260A$$

$$\hat{j}_3 = \hat{j}_4 = \hat{j}_r = -0.45740A$$

$$\hat{v}_4 = \hat{j}_2(R_1 + R_3) + \hat{I}_2 * R_1 = (0.45740A)(5k\Omega + 2k\Omega) + (-1A)(5k\Omega) = -1.7982kV$$

The sensitivities due to each element are

$$\frac{\partial v_o}{\partial R_1} = -\hat{j}_1 j_1 = -(-0.54260)(-177.534 * 10^{-6}) = -96.330\mu V/\Omega$$

$$\frac{\partial v_o}{\partial R_2} = -\hat{j}_2 j_2 = -(0.45740)(6.3229 * 10^{-6}) = -2.8921\mu V/\Omega$$

$$\frac{\partial v_o}{\partial R_3} = -\hat{j}_3 j_3 = -(-0.45740)(187.534 * 10^{-6}) = 85.778\mu V/\Omega$$

$$\frac{\partial v_o}{\partial r} = -\hat{j}_r j_r = -(-0.45740)(3.6771 * 10^{-6}) = 1.6819\mu V/\Omega$$

$$\frac{\partial v_o}{\partial \beta} = \hat{v}_4 j_4 = (-1789.2)(3.6771 * 10^{-6}) = -6.6122mV$$

The maximum deviation of the resistances and β are

$$|\Delta R_1|_{max} = 0.05 * R_1 = 0.05 * 5k\Omega = 250\Omega$$

$$|\Delta R_2|_{max} = 0.05 * R_2 = 0.05 * 200k\Omega = 10k\Omega$$

$$|\Delta R_3|_{max} = 0.05 * R_3 = 0.05 * 2k\Omega = 100\Omega$$

$$|\Delta r|_{max} = 0.05 * r = 0.05 * 500\Omega = 25\Omega$$

$$|\Delta\beta|_{max} = 0.05 * \beta = 0.05 * 50 = 2.5$$

The maximum deviation of the output voltage is

$$|\Delta v_o|_{max} = \left|\frac{\partial v_o}{\partial R_1}\right| |\Delta R_1|_{max} + \left|\frac{\partial v_o}{\partial R_2}\right| |\Delta R_2|_{max} + \left|\frac{\partial v_o}{\partial R_3}\right| |\Delta R_3|_{max} + \left|\frac{\partial v_o}{\partial r}\right| |\Delta r|_{max} + \left|\frac{\partial v_o}{\partial \beta}\right| |\Delta\beta|_{max}$$

$$|\Delta v_o|_{max} = (96.330\mu V/\Omega)*(250\Omega) + (2.8921\mu V/\Omega)*(10k\Omega) + (85.778\mu V/\Omega)*(100\Omega) +$$
$$(1.6819\mu V/\Omega)*(25\Omega) + (6\ 612mV/\Omega)*(2.5)$$
$$= 24.083mV + 28.921mV + 8.578mV + 0.042mV + 16.530mV = 78.154mV$$

$$v_o = j_1 * R_1 = (-177.53\mu A)(5k\Omega) = -0.88767V$$

$$j_{o,max} = -0.88767 + 78.154mA = -0.80952V$$

$$j_{o,min} = -0.88767 - 78.154mV = -0.96582V$$

The range of the output voltage v_o due to sensitivity is

$$-0.96582V \le v_o \le -0.80952V$$