1 a) Assume that the transfer function of the second-order filter is

transducer function

$$H_t(s) = -\frac{\omega_p^2}{s^2 + s\frac{\omega_p}{Q_p} + \omega_p^2}, \text{ then } E(s) = \frac{1}{H_t(s)} = -\frac{s^2 + s\frac{\omega_p}{Q_p} + \omega_p^2}{\omega_p^2}$$

Group delay $\tau_g(\omega) = \mathrm{Re}\left[\frac{dE(s)/ds}{E(s)}\right]_{s=j\omega}$

$$= \mathrm{Re}\left[\frac{2s + \frac{\omega_p}{Q_p}}{s^2 + s\frac{\omega_p}{Q_p} + \omega_p^2}\right]_{s=j\omega}$$

$$\tau_g(\omega_p) = \mathrm{Re}\left[\frac{2j\omega_p + \frac{\omega_p}{Q_p}}{-\omega_p^2 + j\omega_p \cdot \frac{\omega_p}{Q_p} + \omega_p^2}\right]$$

$$= \frac{2Q_p}{\omega_p} = \frac{2 \times 4}{2\,\mathrm{M\,rad/s}} = 4\,\mu s.$$

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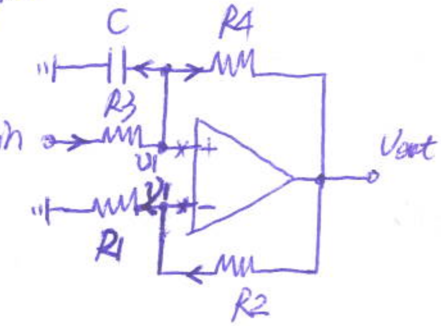
b) $Q_p \gg 1$, $E(s) = -\frac{s^2 + \omega_p^2}{\omega_p^2}$

$\Rightarrow E(j\omega) = -1 + (\omega/\omega_p)^2$, real function of ω.

so $\angle E(j\omega) = 0°$ for all frequencies, then $\tau_g(\omega_p) = 0$.

2. Based on the characteristic of Amplifier and use KCL at each node:



$$\begin{cases} \dfrac{V_{in}-V_1}{R_3} = \dfrac{V_1}{1/SC} + \dfrac{V_1 - V_{out}}{R_4} \\ \dfrac{V_1}{R_1} = \dfrac{V_{out} - V_1}{R_2} \end{cases}$$

$$\Rightarrow H(S) = \frac{V_{out}}{V_{in}} = \frac{(G_1+G_2)G_3}{SG_2C + G_2G_3 - G_1G_4}, \quad G_i = \frac{1}{R_i} \ (i=1,2,3,4)$$

Since $R_1/R_2 = R_3/R_4$, $G_1G_4 = G_2G_3$, thus

$$H(S) = \frac{(G_1+G_2)G_3}{SG_2C}$$

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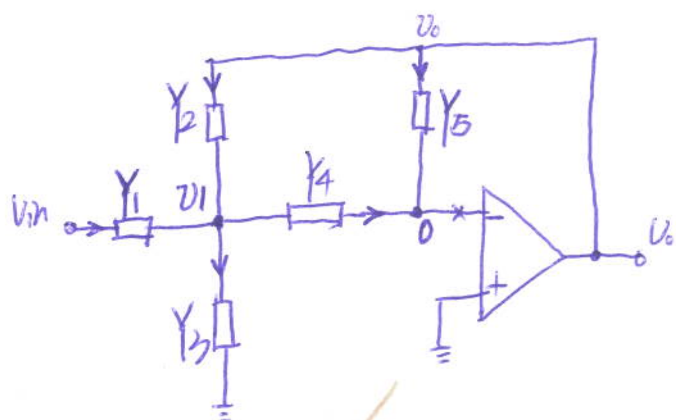
3. a) use KCL at each node:



$$\begin{cases} Y_1(V_{in}-V_1) + Y_2(V_0 - V_1) \\ \quad = Y_3V_1 + Y_4V_1 \\ Y_4V_1 + Y_5V_0 = 0 \end{cases}$$

$$\Rightarrow H(S) = \frac{V_{out}}{V_{in}} = -\frac{Y_1Y_4}{Y_1Y_5 + Y_2Y_4 + Y_2Y_5 + Y_3Y_5 + Y_4Y_5}$$

b) to realize a high-pass filer, H(s) should be in $\dfrac{a_2S^2}{b_2S^2+b_1S+b_0}$ form.

by choosing $Y_1 = SC_1$, $Y_2 = SC_2$, $Y_3 = G_3$, $Y_4 = SC_4$, $Y_5 = G_5$. we could get

$$H(S) = -\frac{SC_1 \times SC_4}{SC_1G_5 + SC_2 \times SC_4 + SC_2 \times G_5 + G_3G_5 + SC_4G_5}$$, satisfying that form.

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