

MIDTERM EXAMINATION

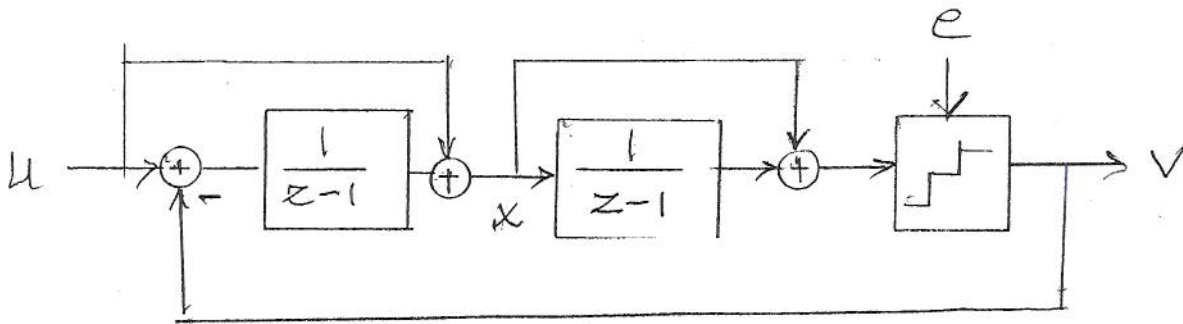
ECE 627

May 15, 2009, 3-3:50 pm

KEAR 305

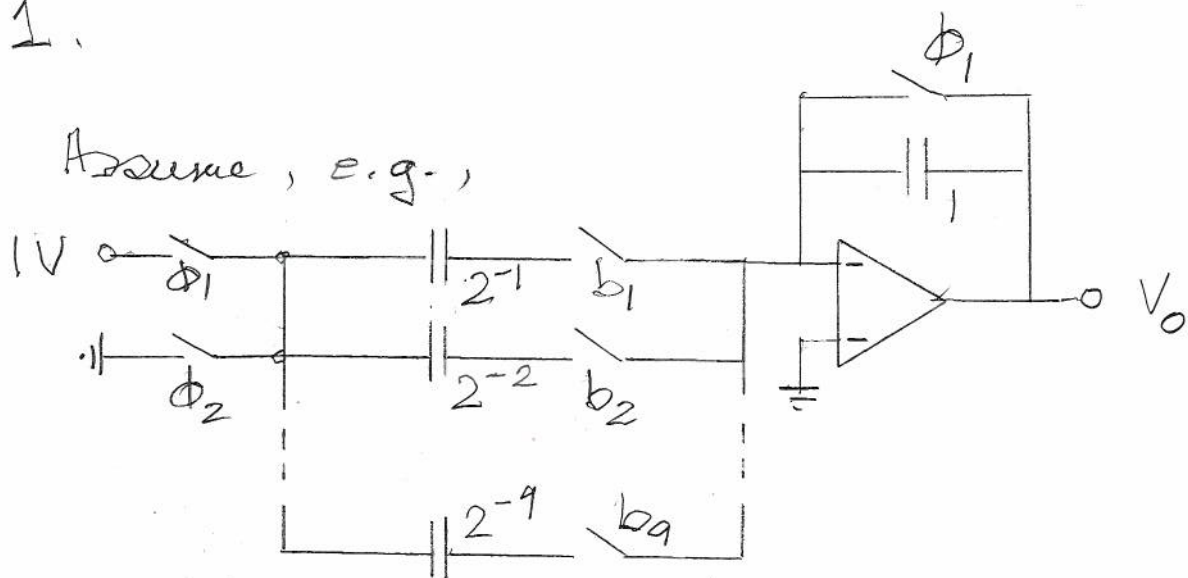
Open book, open notes

1. In an 9-bit DAC constructed from binary-weighted capacitors all capacitors may have a relative error  $r$ . What is the maximum value of  $|r|$  if the DNL must be below  $\frac{1}{2}$  LSB? What is the largest INL for this  $r$ ?
2. What is the restriction on the offset errors of the comparators in an 8-bit flash ADC if no missing codes are allowed? Assume  $V_{ref} = 1$  V.
3. a. Find the NTF and STF of the delta-sigma modulator shown.  
b. Find the value of the internal signal  $X(z)$ .  
c. How do the NTF and STF change if an amplifier of gain  $A$  is placed in front of the quantizer?



# Solutions.

1.



$$V_0(n) = \sum_{i=1}^9 b_i(n) 2^{-i} \quad \text{for ideal caps}$$

If the input caps have relative error  $r_i$

$$V_0(n) = \sum_{i=1}^9 b_i(n) (1+r_i) 2^{-i}$$

DNL max. for 10000...  $\leftrightarrow$  0111..., if  $r_{MSB} \rightarrow (1+r_{max}) 2^{-1}$ , and all others become  $(1-r_{max}) 2^{-i}$ ; then

$$DNL_{max} = r_{max} \sum_{i=1}^9 2^{-i} = r_{max} (1-2^{-9}) \text{ (V)}$$

$$1 \text{ LSB} = 2^{-9} \text{ V, so}$$

$$DNL_{max} \approx 2^9 r_{max} (\text{LSB}) \stackrel{!}{=} 0.5 (\text{LSB})$$

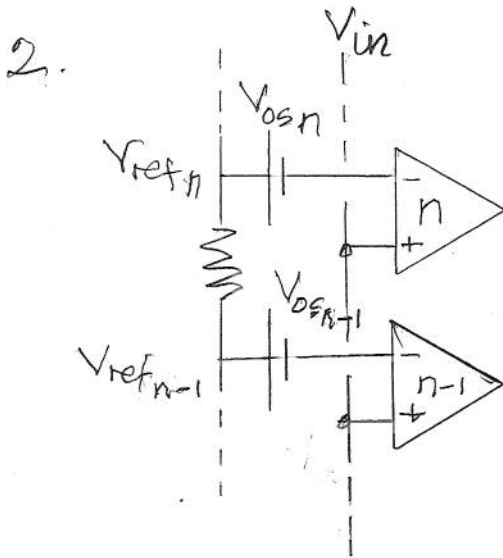
$$\text{So } r_{max} \approx 2^{-10} \approx 0.9766 \times 10^{-3}$$

For absolute INL with  $r_{max}$ , 111...1

$$INL_{max}^a = DNL_{max} \cong 0.5 \text{ (LSB)} \cong 0.977 \mu\text{V}$$

Endpoint DNL max occurs for 1000...0

$$INL_{max}^e = 2^{-1} r_{max} \cong 0.488 \mu\text{V} \cong 0.25 \text{ LSB}$$



The ref. input of comparator  $n$  must be higher than that of comp.  $n-1$ . So

$$V_{refn} - V_{osn} > V_{refn-1} - V_{osn-1}$$

For worst case,  $V_{osn} = +V_{osmax}$  and

$V_{osn-1} = -V_{osmax}$ . This gives

$$2V_{osmax} < V_{LSB} = 2^{-8} V \approx 3.9 \text{ mV}$$

$$V_{osmax} \approx 1.953 \text{ mV}$$

$$3.a \quad X = U + I(U - V), \text{ where } I = 1/(z-1)$$

$$V = E + X(1+I)$$

$$V = [E + (I+1)^2 U] / [1 + I(I+1)]$$

$$STF = \frac{1}{1 - z^{-1} + z^{-2}}, \quad NTF = \frac{(1 - z^{-1})^2}{1 - z^{-1} + z^{-2}}$$

$$b. \quad X = U + I(U - V) = \frac{1 - z^{-1}}{1 - z^{-1} + z^{-2}} [U - z^{-1}E]$$

c.  $X$  as before.

$$V = E + AX(1+I) = \frac{AU + (1 - z^{-1})^2 E}{1 + (A-2)z^{-1} + z^{-2}}$$

$$STF = \frac{A}{1 + (A-2)z^{-1} + z^{-2}}$$

$$NTF = \frac{(1 - z^{-1})^2}{1 + (A-2)z^{-1} + z^{-2}}$$