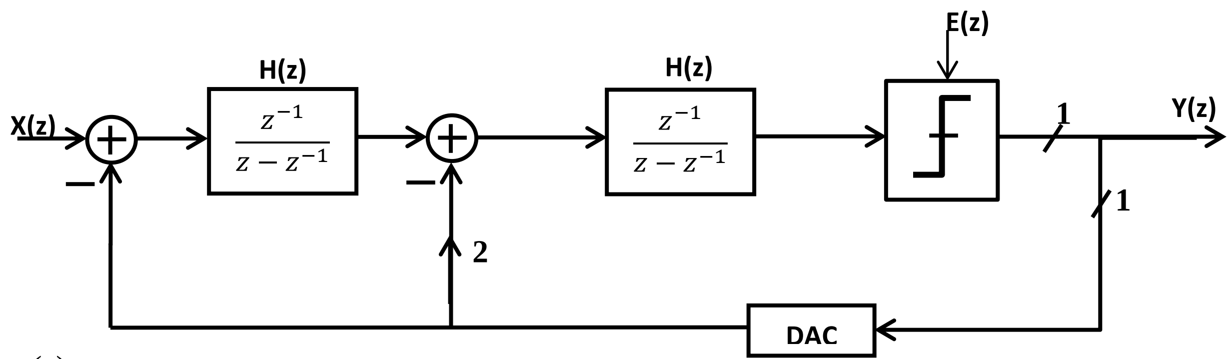
ECE-627 MIDTERM

May 22, 2001
Open Book

Problem-1: For the ΔΣ ADC shown below:

a. Find the NTF and STF.
b. Show the zeros and poles of NTF in the z-plane and draw the frequency response of the NTF.
c. Do you expect the loop to be stable? Why or Why not?



Part (a)

$Y = E + H * [H * (X-Y) - 2Y]$

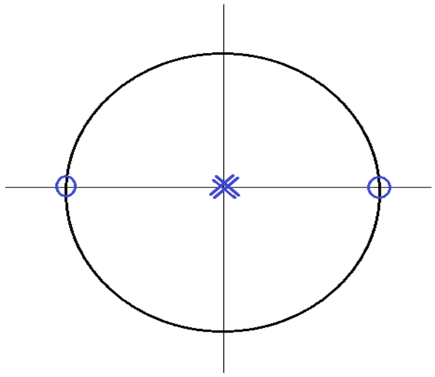
$\{H^2 + 2H + 1\} Y = (H+1)^2 Y = H^2 X + E$

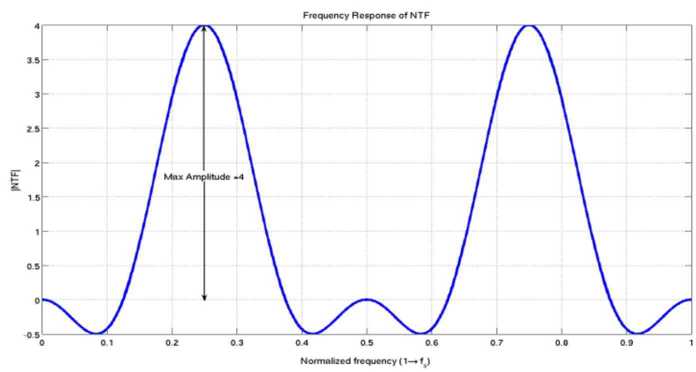
$H = \frac{z^{-2}}{1-z^{-2}}$; $(H+1)^2 = \frac{1}{(1-z^{-2})^2}$

$Y = z^{-4} X + (1 - z^{-2})^2 E$

So; STF $= z^{-4}$ and NTF $= (1 - z^{-2})^2$

Part (b)
Poles(4) at z=0 ; zeros (2) at z=1 and z=-1





Pole zero plot and Frequency Response

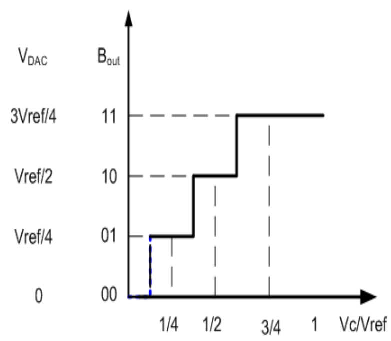
Part(c)

Yes, Since it can be obtained by mapping z with z^2. Which form a stable 2^{nd}-order loop.

Problem-2: The STF of a ΔΣ ADC is z^{-2}; its NTF is

$$H_N(z) = \frac{(1 - z^{-1})^2}{1 - 0.1z^{-1}}$$

It uses a 2-bit internal quantizer, with the characteristics shown below, and V_{ref} = 1V. Assuming a dc input, what is the input voltage range which guarantees even in the worst case that the quantizer is not overloaded ?



Solution:

Suppose the signal appearing at the input port of the 2-bit quantizer is $v_c(n)$,
Then

V_c=Y(z)-E(z)
=X STF +E [NTF-1]

So

$V_c(z)\,(1\text{-}0.1z^{-1}) = X(z)\, z^{-2}(1\text{-}0.1z^{-1}) + E(z)[\,(1\text{-}z^{-1})^2\text{-}(1\text{-}0.1z^{-1})]$
$= X(z)\, z^{-2}(1\text{-}0.1z^{-1}) + E(z)\,(z^{-2}\text{-}\ 1.9\, z^{-1})$

Finally we get,

$V_c(z)\,(1\text{-}0.1z^{-1}) = X(z)\, z^{-2}(1\text{-}0.1z^{-1}) + E(z)\,(z^{-2}\text{-}\ 1.9\, z^{-1}) + V_c(z)\,0.1z^{-1}$

in time domain

$v_c(n) = 0.9\,x + e(n\text{-}2) - 1.9\, e(n\text{-}1) + 0.1\, v_c(n\text{-}1)$
$= 0.9\,x + e(n\text{-}2) - 2\, e(n\text{-}1) + 0.1\, y(n\text{-}1)$

to ensure the quantizer is not overloaded,

$$-\frac{V_{LSB}}{2} \le\ v_c(n) \le V_{ref} - \frac{V_{LSB}}{2}$$

$$-0.125 \le\ v_c(n) \le 0.875$$

and keep in mind that

$$-\frac{V_{LSB}}{2} \le\ e(n) \le \frac{V_{LSB}}{2}$$
$$-0.125 \le\ e(n) \le 0.125$$

that means in the worst case

$0.9x \leq \left. v_c(n)\right|_{max} - \left. e(n-2)\right|_{max} + 1.9 \left. e(n-1)\right|_{min} - 0.1 \left. v_c(n-1)\right|_{max}$
= 0.875 -0.125+1.9x(-0.125)-0.1x0.875
= 0.425

and
$0.9x \leq \left. v_c(n)\right|_{max} - \left. e(n-2)\right|_{max} + 1.9 \left. e(n-1)\right|_{min} - 0.1 \left. v_c(n-1)\right|_{max}$
= 0.125 - (-0.125) +1.9x(-0.125)-0.1x(-0.125)
= 0.25

Eventually we get

$$0.2778 \leq x \leq 0.4722$$