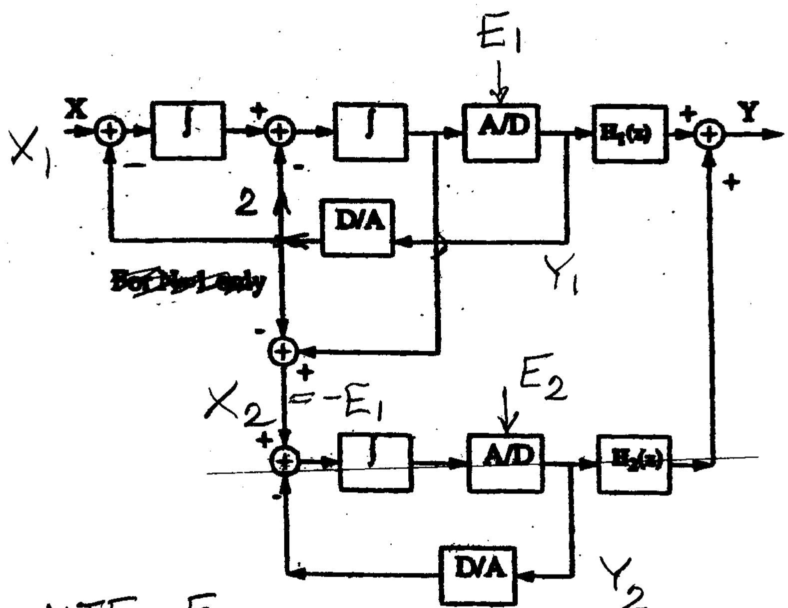
# HOMEWORK 5

WHAT SHOULD BE THE TRANSFER FUNCTIONS OF H1 AND H2 IN THE 2+1 MASH SHOWN BELOW SO THAT THE INPUT X APPEARS IN Y UNDISTORTED, AND THE QUANTIZATION ERROR OF THE FIRST STAGE IS CANCELLED? WHAT ARE THE NTF AND STF FOR THE WHOLE SYSTEM?

ASSUME THAT THE TRANSFER FUNCTION OF THE INTEGRATORS IS $I(z) = \frac{z^{-1}}{1-z^{-1}}$.




$$Y_1 = STF_1 \cdot X_1 + NTF_1 \cdot E_1$$

$$Y_2 = STF_2 \cdot (-E_1) + NTF_2 \cdot E_2$$

$$Y_1 = E_1 + I\left[-2Y_1 + I(X_1 - Y_1)\right] = E_1 + I^2 X_1 - (2I + I^2)Y$$

$$Y_1 = (1+I)^{-2}(E_1 + I^2 X_1) = z^{-2}X_1 + (1-z^{-1})^2 E_1$$

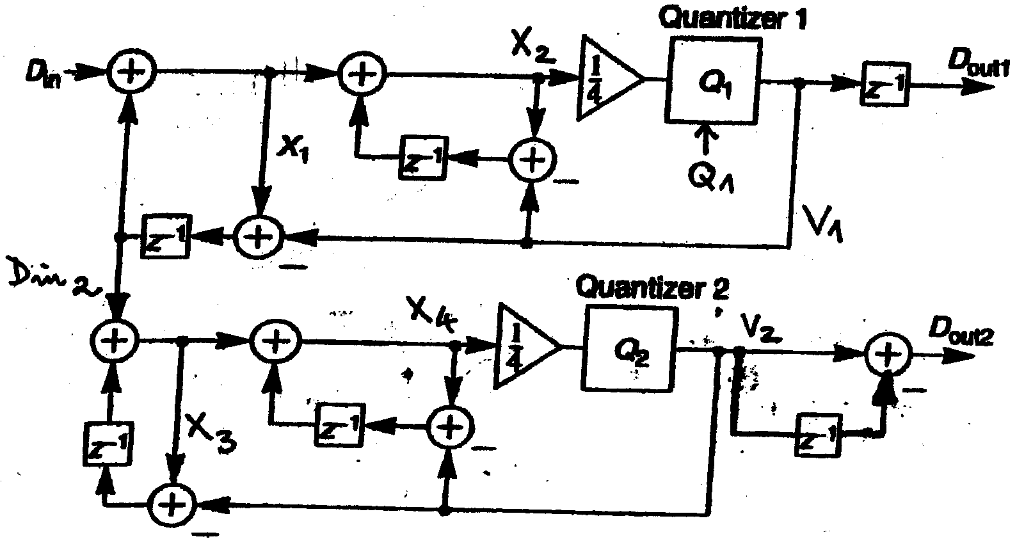
$$Y_2 = E_2 - I(E_1 + Y_2) \rightarrow Y_2 = -z^{-1}E_1 + (1-z^{-1})E_2$$

$$Y = H_1\left[z^{-2}X_1 + (1-z^{-1})^2 E_1\right] + H_2\left[-z^{-1}E_1 + (1-z^{-1})E_2\right]$$

$$H_1(1-z^{-1})^2 - H_2 z^{-1} = 0 \rightarrow H_1 = z^{-1},\ H_2 = (1-z^{-1})^2$$

$$Y_1 = z^{-3}X_1 + (1-z^{-1})^3 E_2$$

. CALCULATE THE OUTPUTS Dout1 AND Dout2 IN TERMS OF THE INPUT Din AND HE QUANTIZATION ERRORS e1 AND e2 FOR THE MASH D/A CONVERTER SHOWN ELOW IN FIG.1.




$$\begin{cases} x_1 = Din + z^{-1}(x_1 - V_1) & \Rightarrow \quad x_1 = \dfrac{Din - z^{-1}V_1}{1 - z^{-1}} \\ x_2 = x_1 + z^{-1}(x_2 - V_1) & \Rightarrow \quad x_2 = \dfrac{x_1 - z^{-1}V_1}{1 - z^{-1}} \\ V_1 = \frac{1}{4} x_2 + Q_1 & \end{cases}$$

$$\Rightarrow V_1 = \frac{1}{4}\left\{ \frac{1}{(1-z^{-1})^2} Din + \frac{-z^{-1} - z^{-1} + z^{-2}}{(1-z^{-1})^2} V_1 \right\} + Q_1 = \frac{Din}{4(1-z^{-1})^2} + \frac{z^{-1}(z^{-1}-2)}{4(1-z^{-1})^2} V_1 + Q_1$$

$$\Rightarrow V_1 \underbrace{\left( 4(1-z^{-1})^2 + z^{-1}(2-z^{-1}) \right)}_{4 - 6z^{-1} + 3z^{-2}} = Din + 4(1-z^{-1})^2 Q_1$$

$$\Rightarrow \boxed{V_1 = \frac{Din}{4-6z^{-1}+3z^{-2}} + \frac{4(1-z^{-1})^2}{4-6z^{-1}+3z^{-2}} Q_1}$$

$$Dout_1 = z^{-1} \times V_1$$

$$Din_2 = z^{-1}(x_1 - V_1) = z^{-1} \frac{Din - z^{-1}V_1}{1-z^{-1}} - z^{-1}V_1 = \frac{z^{-1}}{1-z^{-1}} (Din - V_1)$$

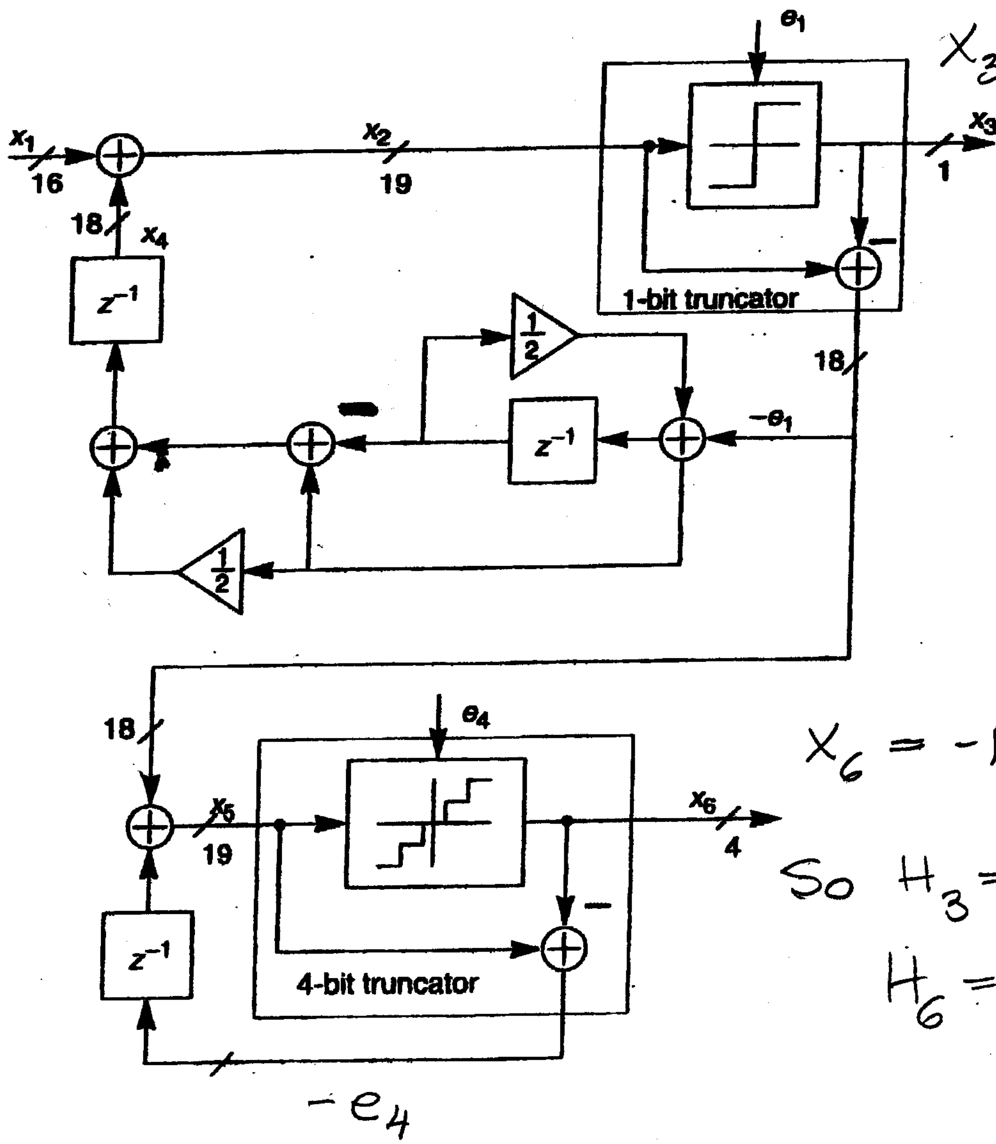
$$V_2 = \frac{Din_2}{4-6z^{-1}+3z^{-2}} + \frac{4(1-z^{-1})^2}{4-6z^{-1}+3z^{-2}} Q_2 =$$

$$= \frac{z^{-1}}{1-z^{-1}} \times \frac{1}{\underbrace{4-6z^{-1}+3z^{-2}}_{\text{"}4-\dots\text{"}}} Din - \frac{z^{-1}}{1-z^{-1}} \times \frac{1}{(4-\dots)^2} Din - \frac{z^{-1}}{1-z^{-1}} \times \frac{4(1-z^{-1})^2}{(4-\dots)^2} Q_1 + \frac{4(1-z^{-1})^2}{(4-\dots)} Q_2$$

$$\& \quad Dout_2 = (1-z^{-1}) V_2$$

$$\Rightarrow \boxed{Dout_2 = \frac{3z^{-1}(1-z^{-1})^2}{(4-6z^{-1}+3z^{-2})^2} Din - \frac{4z^{-1}(1-z^{-1})^2}{(4-6z^{-1}+3z^{-2})^2} Q_1 + \frac{4(1-z^{-1})^3}{4-6z^{-1}+3z^{-2}} Q_2}$$

. ANALYZE THE CIRCUIT OF FIG.2. WHAT ADDITIONAL TRANSFER FUNCTIONS ARE NEEDED FOR NOISE CANCELLATION BEFORE x3 AND x6 CAN BE COMBINED?




$$X_3 = X_1 + \frac{(1-z^{-1})^2 E_1}{1-z^{-1}/2}$$

$$X_6 = -E_1 + (1-z^{-1})E_4$$

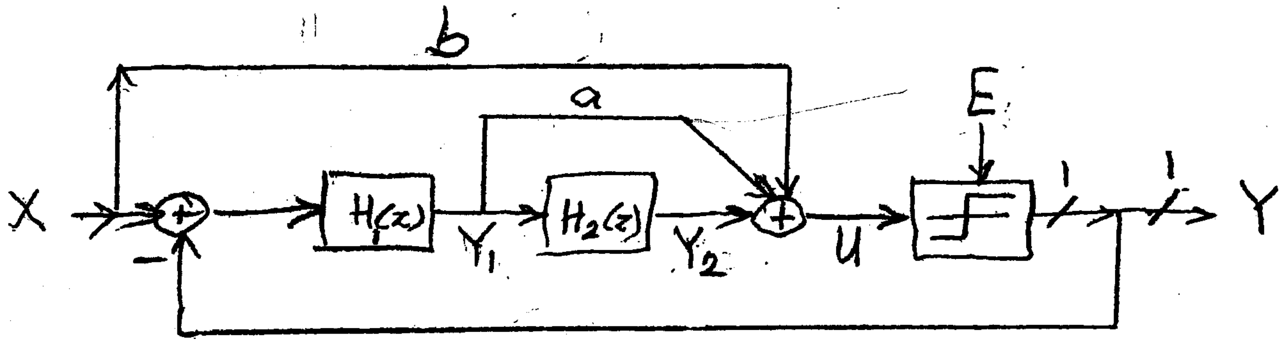
So $H_3 = 1$

$$H_6 = 2\,\frac{(1-z^{-1})^2}{2-z^{-1}}$$

FIG.2

**2(a)** For the feedforward delta-sigma loop, find $b$ such that the signal transfer function is $1$.

**2(b)** Find $a$ such that for $H_1(z) = H_2(z) = \dfrac{1}{z-1}$, the noise transfer function is $(1-z^{-1})^2$.




$$H_1(z) = H_2(z) = \frac{z^{-1}}{1-z^{-1}}$$

$$Y_1 = H_1(X-Y), \quad Y_2 = H_1 H_2 (X-Y)$$

$$U = bX + aY_1 + Y_2 = bX + (a+H_2)H_1(X-Y)$$
$$= [b+(a+H_2)H_1]X - (a+H_2)H_1 Y = Y - E$$

$$[1+(a+H_2)H_1]\,Y = [b+(a+H_2)H_1]\,X + E \;; \quad b \to 1 \text{ for undistorted } X$$

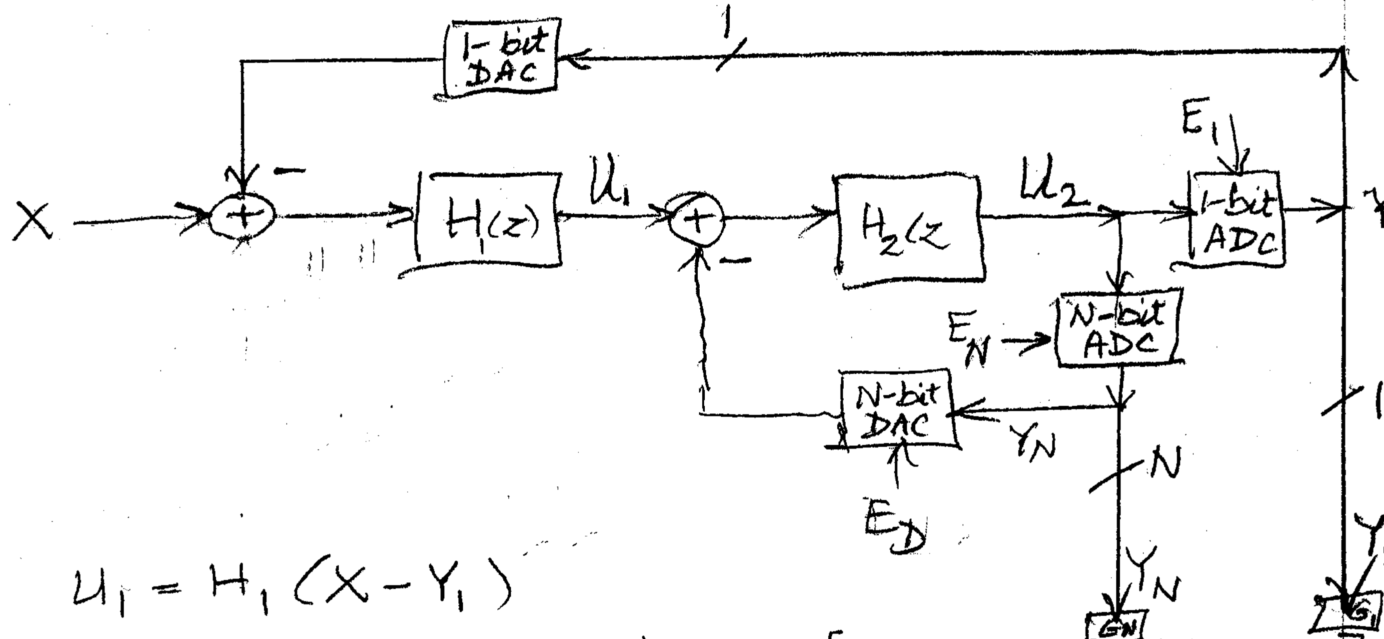
$$Y = X + \frac{E}{[\;]} \qquad \frac{1}{[\;]} = \frac{1}{1+aH_1+H_1^2} = \frac{(1-z^{-1})^2}{(1-z^{-1})^2 + az^{-1}(1-z^{-1}) + z^{-2}}$$

Denom. is

$$1 - \cancel{2z^{-1}} + \cancel{z^{-2}} + \cancel{az^{-1}} - \cancel{az^{-2}} + \cancel{z^{-2}}$$

For $a = 2$, Denom $= 1$.

# $2^{nd}$-order Hairapetian




$$U_1 = H_1(X - Y_1)$$

$$U_2 = H_2(U_1 - Y_N - E_D) = H_2[H_1 X - H_1 Y_1 - Y_N - E_D]$$

$$Y_1 = U_2 + E_1 = H_1 H_2 X - H_1 H_2 Y_1 - H_2 Y_N - H_2 E_D + E_1$$

$$Y_N = U_2 + E_N = Y_1 - E_1 + E_N$$

$$Y_1[1 + H_1 H_2 + H_2] = H_1 H_2 X + (H_2 + 1)E_1 - H_2 E_N - H_2 E_D$$

$$Y_N[\qquad\qquad] = Y_1[\;] - E_1[\;] + E_N[\;] =$$

$$Y_N = H_1 H_2 X - E_1 H_1 H_2 + E_N(1 + H_1 H_2) - H_2 E_D$$

To cancel $E_1$, multiply $Y_1$ by $G_1$ & $Y_N$ by $G_N$

$$G_1(H_2+1) \stackrel{!}{=} G_N H_1 H_2, \quad G_1 \stackrel{!}{=} F H_1 H_2, \quad G_N = F(H_2+1)$$

To transmit $X$ undistorted, the factor of $X$ in $G_1 Y_1 + G_N Y_N$ must be $z^{-k}$.

Overall STF

$$(G_1 + G_N)\frac{H_1 H_2}{1 + H_1 H_2 + H_2} = F H_1 H_2, \quad \text{so } F \stackrel{!}{=} z^{-k}/(H_1 H_2)$$

$$G_1 = z^{-k}, \quad G_N = z^{-k}\frac{H_2 + 1}{H_1 H_2}$$