Multi-Stage (MASH) Modulators

ECE 627

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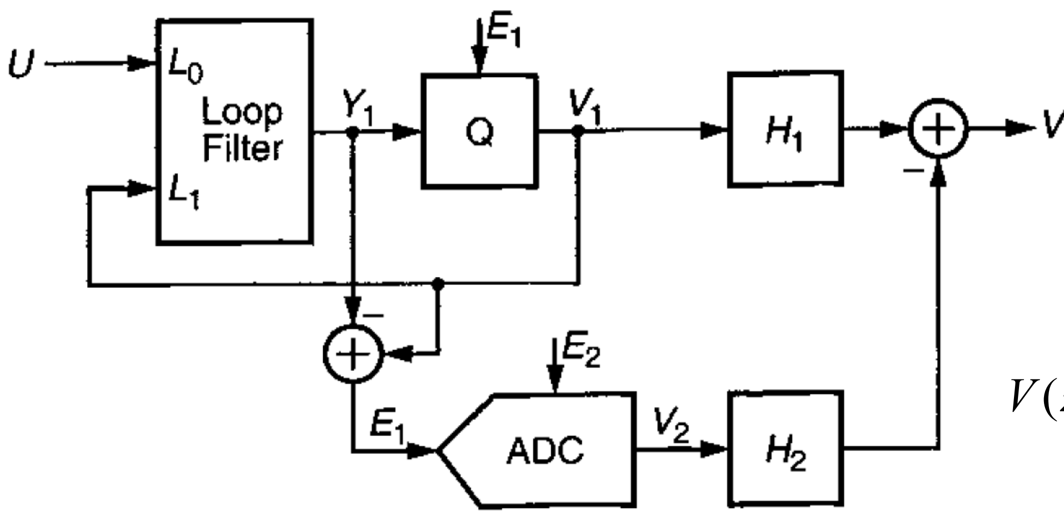
Outline

- Leslie-Singh (L-0 Cascade) Structure
- Cascade (MASH) Structure
  - Two Stage
  - Three Stage
- Noise Leakage in Cascade Modulators

Leslie-Singh (L-0) Cascade Structure



$$V(z) = H_1(z) \cdot V_1(z) - H_2(z) \cdot V_2(z)$$

$$H_1(z) = z^{-k} \qquad H_2(z) = NTF_1$$

$$V(z) = z^{-k}[STF_1(z)U(z) - NTF_1(z)E_2(z)]$$

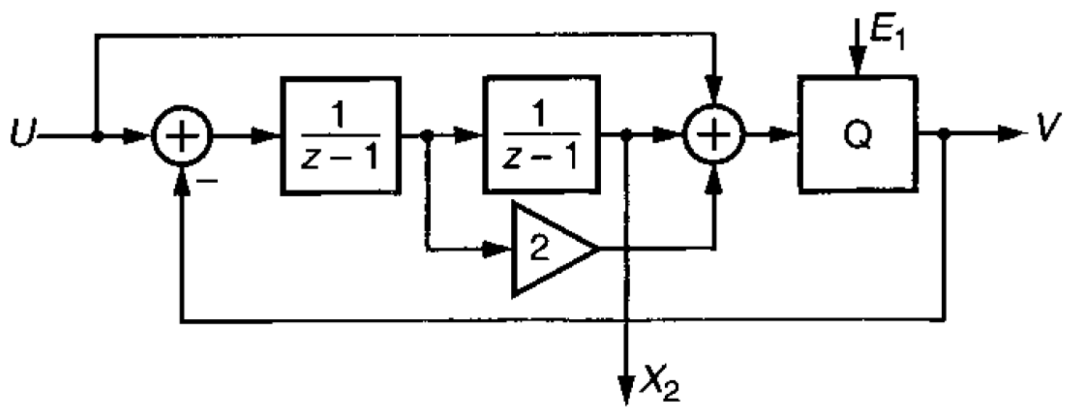
- Quantization error of first stage, E_1, is feed to a multi-bit (e.g. 10 bit) ADC; V_1 and V_2 are filtered by H_1 and H_2, and added
- E_1 can be cancelled, only remaining E_2 is noise shaped
- Usually $E_2 << E_1$ because ADC is outside the loop thus can be made much cheaper and simpler

Leslie-Singh (L-0) Cascade Structure

- Quantizer must be delay-free (not practical) or Y_1 must be delayed for successful subtraction
- Alternatively, use Y_1 as input to ADC, then

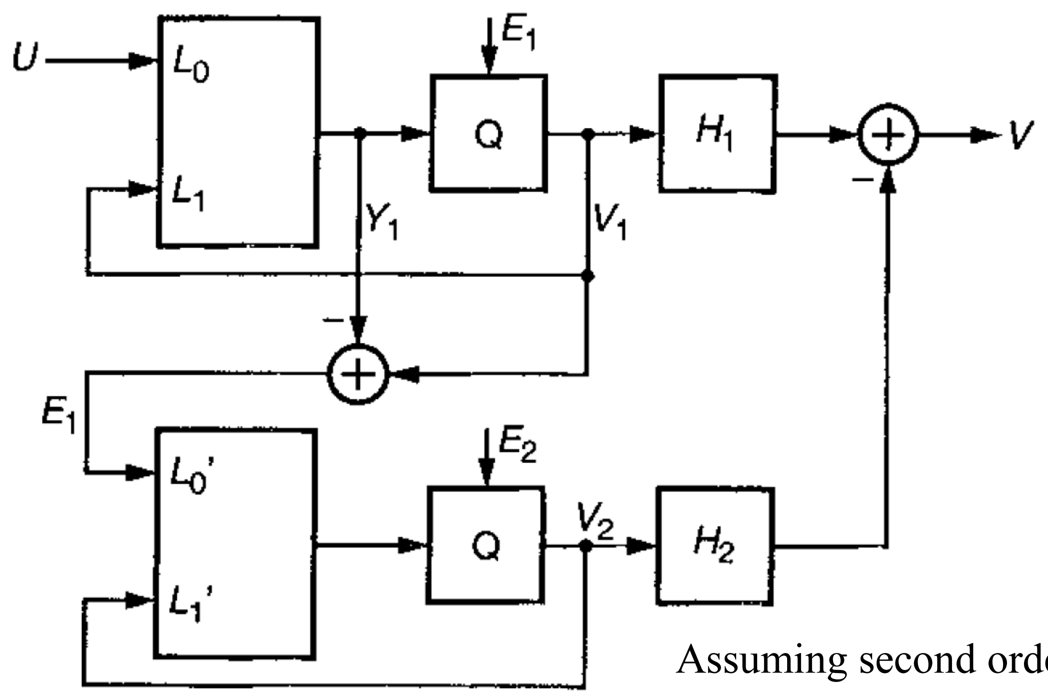
$$V(z) \approx z^{-k}[STF_1(z)U(z) - NTF_1(z)\,E_2(z)] \text{ for } |\text{NTF}| << 1$$

+ No subtraction
- Larger input to second stage since both U and E_1 is fed to ADC
- Low distortion stage (e.g. CIFF) should be used as first MASH



$$X_2(z) = -z^{-2}E_1$$

Cascade (MASH) Structure



$$V_1(z) = STF_1(z) \cdot U(z) + NTF_1(z) \cdot E_1(z)$$

$$V_2(z) = STF_2(z) \cdot E_1(z) + NTF_2(z) \cdot E_2(z)$$

$$V(z) = H_1(z) \cdot V_1(z) + H_2(z) \cdot V_2(z)$$

With $H_1 = STF_2$ and $H_2 = NTF_1$

$$H_1(z) \cdot NTF_1(z) - H_1(z) \cdot NTF_2(z) = 0$$

Assuming second order loop where $STF = z^{-2}$ and $NTF = (1 - z^{-1})^2$

Then $V(z) = z^{-4} \cdot U(z) - (1 - z^{-1})^4 \cdot E_2(z) = 0$

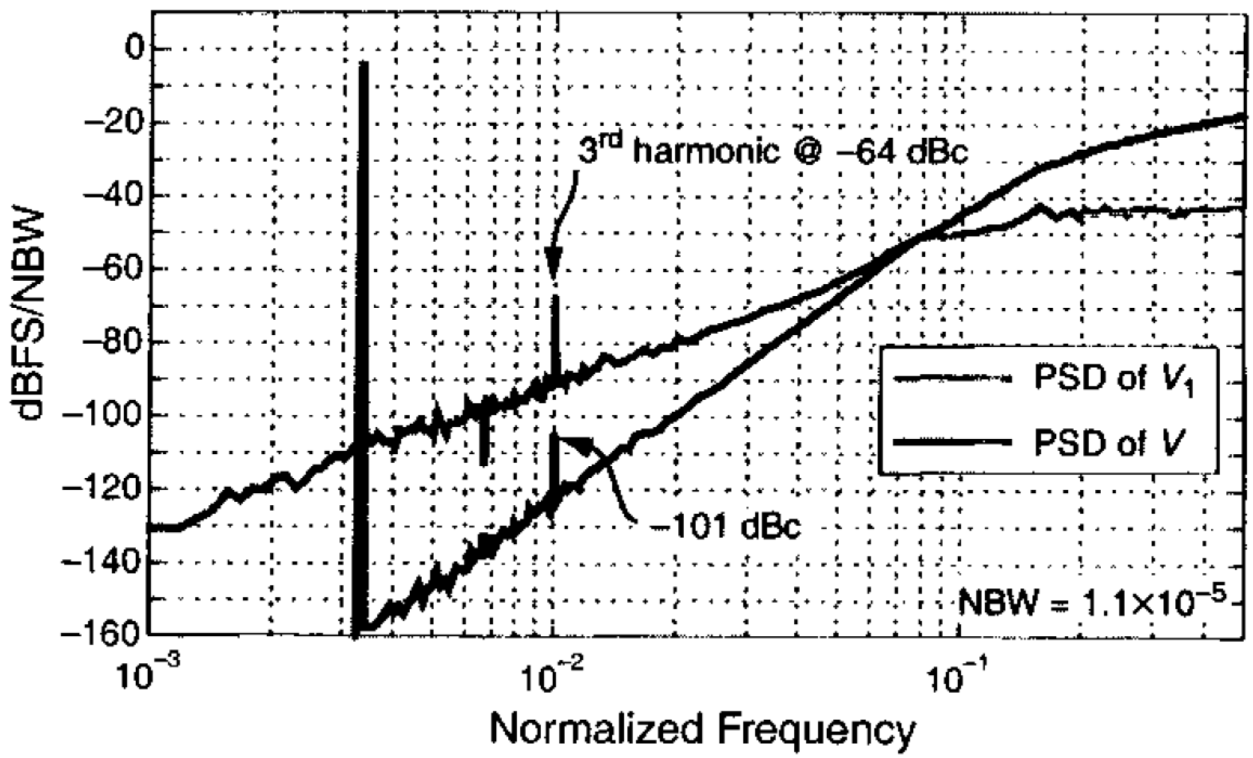
- Noise shaping of a fourth order single-loop converter is obtained with the stability of second order one
- E_1 input to second modulator needs to be scaled in practice

Cascade (MASH) Structure

- + Higher order noise shaping performance achieved without feedback instability problems
- + A multi-bit quantizer can be used in the second stage without any DAC linearity correction because errors of the second stage is multiplied by high-pass H_2 before added to the output
- + Remaining E_2 is more similar to white noise because it is based on a noise-like input of E_1
- + No harmonic distortion of the signal is generated in the second stage

Cascade (MASH) Structure

+ Noise tone reduction
+ Less likely to need dithering

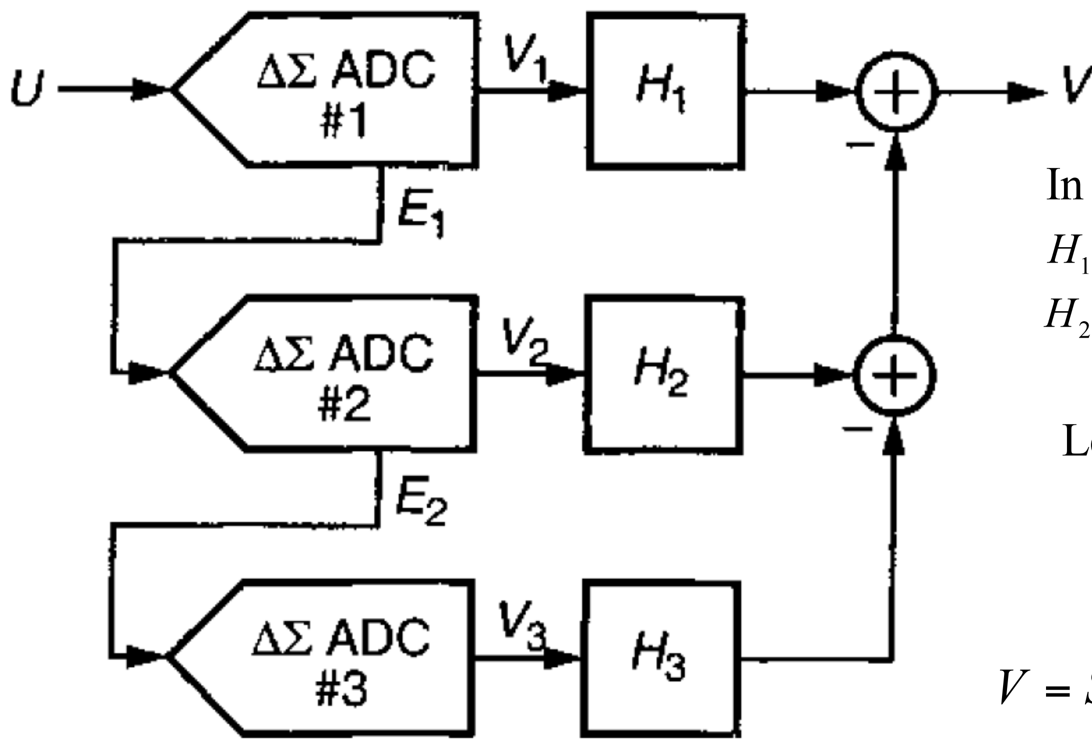


Output spectra of a 2-2 MASH showing 3rd harmonic tone reduction in V

Cascade (MASH) Structure

- Imperfection of analog transfer functions (noise leakage) may result serious SNR deterioration – more on this later
- Higher amplifier open loop gain and slew rate requirements
- More sensitive to capacitor mismatches

3-Stage MASH



$$V = H_1 \cdot V_1 - H_2 \cdot V_2 + H_3 \cdot V_3$$

In order to have:

$$H_1(z) \cdot NTF_1(z) - H_2(z) \cdot STF_2(z) = 0$$

$$H_2(z) \cdot NTF_2(z) - H_3(z) \cdot STF_3(z) = 0$$

Let: $H_1 = STF_2 \cdot STF_3$

$H_2 = NTF_1 \cdot STF_3$

$H_3 = NTF_1 \cdot NTF_2$

$$V = STF_1 \cdot H_1 \cdot U - NTF_3 \cdot H_3 \cdot E_3$$

$$V = STF_1 \cdot H_1 \cdot U + \left(\frac{H_1 \cdot NTF_1 \cdot NTF_2 \cdot NTF_3}{STF_2 \cdot STF_3} \right) E_3$$

3-Stage MASH

- H_1 and STFs usually contains only delays
- Quantization errors of first two sections are cancelled ideally
- Remaining quantization error is filtered by the product of three NTFs
- Highly sensitive to mismatches among analog and digital transfer functions

Noise Leakage in Cascaded Structures

- Large SQNR - obtained by cancelling quantization noise of first (n-1) stages
- Example: 2-Stage MASH
  - Perfect cancellation of 1st Stage quantization noise requires:

    $$H_1 \cdot NTF_1 - H_2 \cdot STF_2 = 0$$

  - H_1 and H_2 are digital blocks
  - NTF_1 and STF_2 are dependent more on non-idealities
  - Perfect cancellation is difficult

Noise Leakage Estimation

- Simulation is the most reliable technique
- Approximate simple results can be obtained
- Example: 3-Stage MASH

$$H_{l1} = H_1 \cdot NTF_1 - H_2 \cdot STF_2 \longrightarrow \text{TF from } E_1 \text{ to V}$$

$$H_{l2} = H_2 \cdot NTF_2 - H_3 \cdot STF_3 \longrightarrow \text{TF from } E_2 \text{ to V}$$

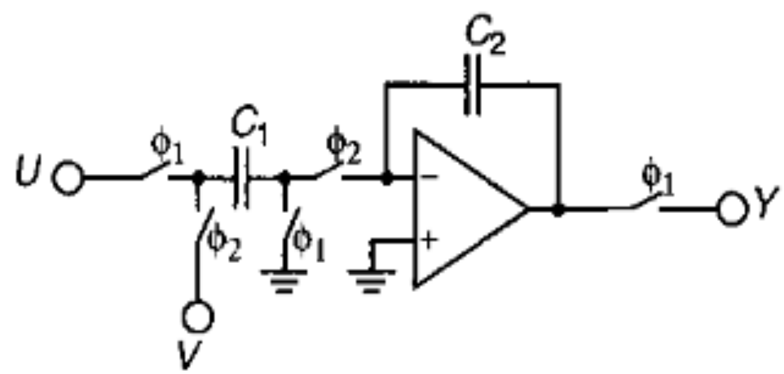
- Ideally $H_{l1}=H_{l2}=0$
- Goal is to estimate effect and values of H_{l1} and H_{l2}

Noise Leakage Estimation Continued…

- Leakage of E_2 is less important
  - H_{l2} terms represent higher order noise shaping than in H_{l1}
  - E_2 smaller than E_1 when multi-bit 2nd stage quantizer is used
  - Thus $|H_{l2}|$ is less important
- Leakage from E_1
  - Imperfect NTF_1 dominates imperfect STF_2, since error in STF_2 are noise shaped
  - If $STF_2=H_1=1$ then $|H_{l1}| \approx |NTF_1 - H_2| = |NTF_{1a} - NTF_{1i}|$
  - But $NTF_1 = 1/(1-L_1)$ and $|L_1| >> 1$(for both actual and ideal)
  - This implies $|H_{l1}| \approx |1/L_{1i} - 1/L_{1a}|$

Noise Leakage Estimation Continued…

- 1-1-1 MASH. D => error in (C_1/C_2), A => DC loop gain
- ideal loop filter TF $I_i(z) = \frac{a}{z-1}$
- actual TF $I_a(z) = \frac{a'}{z-p'}$
- for D << 1 and a/A << 1
- $a' \approx a[1 - D - (1+a)/A]$ and $p' \approx 1 - a/A$
- $L_1(z) = -I(z)$
- $|H_{l1}| = |(1/A) + (z-1)\cdot[D/a + (1+1/a)/A]|$



Noise Leakage Estimation Continued…

- Total Leakage = unfiltered + filtered
- High gain => small E_1/A => Low unfiltered leakage
- D << 1 for low OSR => need good capacitor matching => low value linear filtered noise
- Similarly for a 2-0 MASH it can be shown that
  - Unfiltered noise = $1/A^2$
  - Linearly filtered term = $[1/a_1 + 1/a_2]/A$
  - quadratically filtered term

    $$1/(a_1 \cdot a_2) - 1 + 2[1 - 1/(a_1 \cdot a_2) - 1/a_2^2]/A + 2D/(a_1 \cdot a_2)$$
- Linear and quadratic terms dominate

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