CS 331: Artificial Intelligence
Adversarial Search

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Games we will consider

- Deterministic
- Discrete states and decisions
- Finite number of states and decisions
- Perfect information i.e. fully observable
- Two agents whose actions alternate
- Their utility values at the end of the game are equal and opposite (we call this zero-sum)

"It's not enough for me to win, I have to see my opponents lose"

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Which of these games fit the description?

Two-player, zero-sum, discrete, finite, deterministic games of perfect information

What makes games hard?

- Hard to solve e.g. Chess has a search graph with about 10^{40} distinct nodes
- Need to make a decision even though you can't calculate the optimal decision
- Need to make a decision with time limits

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Formal Definition of a Game

A quintuplet (S, I, Succ(), T, U):

S	Finite set of states. States include information on which player's turn it is to move.
I	Initial board position and which player is first to move
Succ()	Takes a current state and returns a list of (move,state) pairs, each indicating a legal move and the resulting state
T	Terminal test which determines when the game ends. Terminal states: subset of S in where the game has ended
U	Utility function (aka objective function or payoff function): maps from terminal state to real number

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Nim

Many different variations. We'll do this one.

- Start with 9 beaver logos
- In one player's turn, that player can remove 1, 2 or 3 beaver logos
- The person who takes the last beaver logo wins

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Nim

Formal Definition of Nim

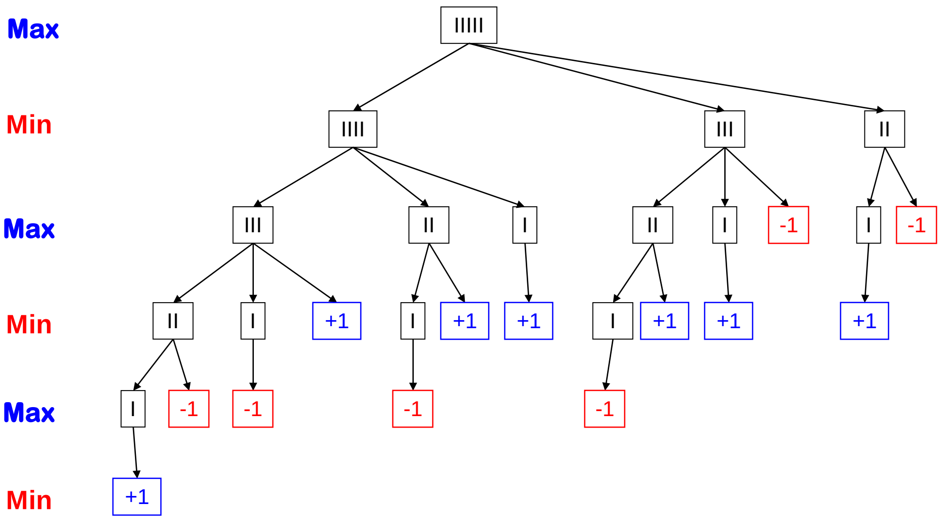
Notation: Max(IIIII)

Who's move # matches left

A quintuplet (S, I, Succ(), T, U):

S	Max(IIIII), Max(III), Max(II), Max(I) Min(IIII), Min(III), Min(II), Min(I)
I	Max(IIIII)
Succ()	Succ(Max(IIIII)) = {Min(IIII),Min(III),Min(II)} Succ(Min(IIII)) = {Max(III),Max(II),Max(I)} Succ(Max(III)) = {Min(II),Min(I)} Succ(Min(III)) = {Max(II),Max(I)} Succ(Max(II)) = {Min(I)} Succ(Min(II)) = {Max(I)}
T	Max(I), Max(II), Max(III), Min(I), Min(II), Min(III)
U	Utility(Max(I) or Max(II) or Max(III)) = +1, Utility(Min(I) or Min(II) or Min(III)) = -1

Nim Game Tree



We'll call the players Max and Min, with Max starting first

How to Use a Game Tree

- Max wants to maximize his utility
- Min wants to minimize Max's utility
- Max's strategy must take into account what Min does since they alternate moves
- A move by Max or Min is called a ply

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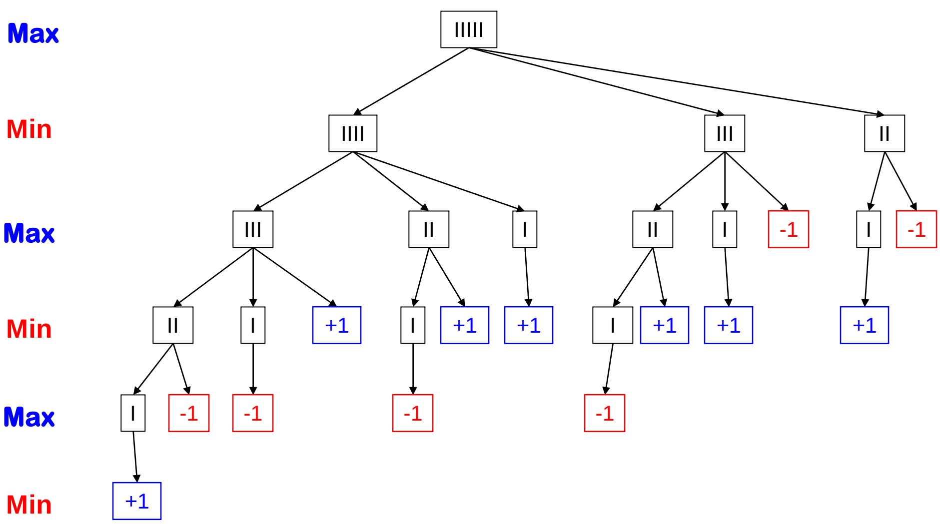
The Minimax Value of a Node

The minimax value of a node is the utility for MAX of being in the corresponding state, *assuming that both players play optimally* from there to the end of the game

$$\text{MINIMAX-VALUE}(n) = \begin{cases} \text{UTILITY}(n) & \text{If n is a terminal state} \\ \max_{s \in Successors(n)} \text{MINIMAX-VALUE}(s) & \text{If n is a MAX node} \\ \min_{s \in Successors(n)} \text{MINIMAX-VALUE}(s) & \text{If n is a MIN node} \end{cases}$$

Minimax value maximizes worst-case outcome for MAX

Nim Game Tree



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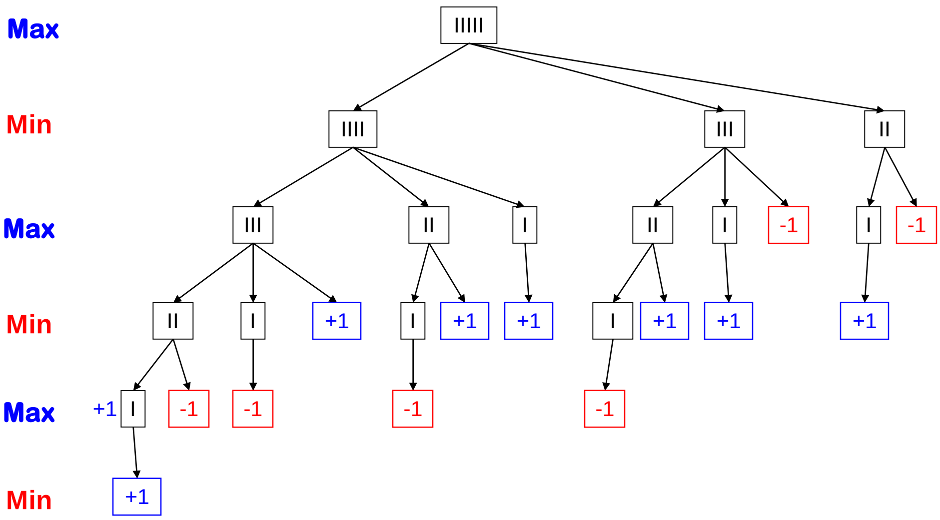
Minimax Values in Nim Game Tree



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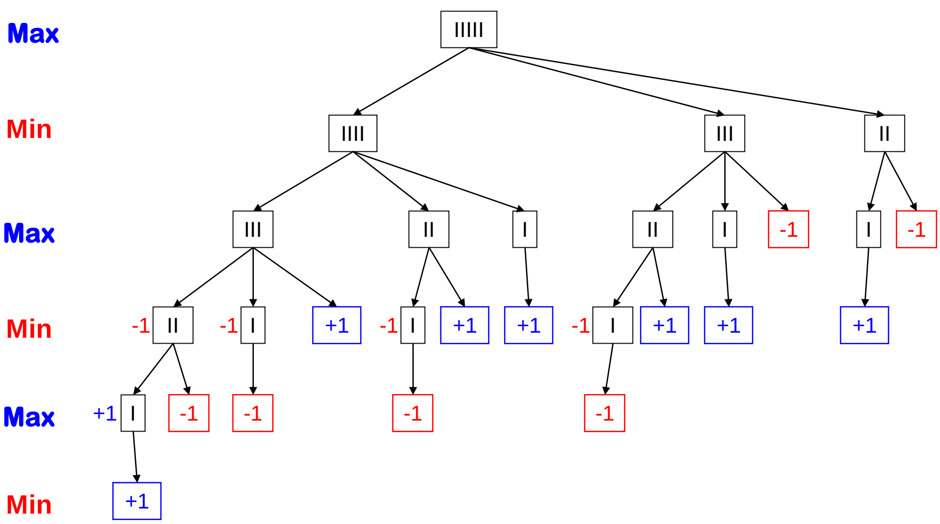

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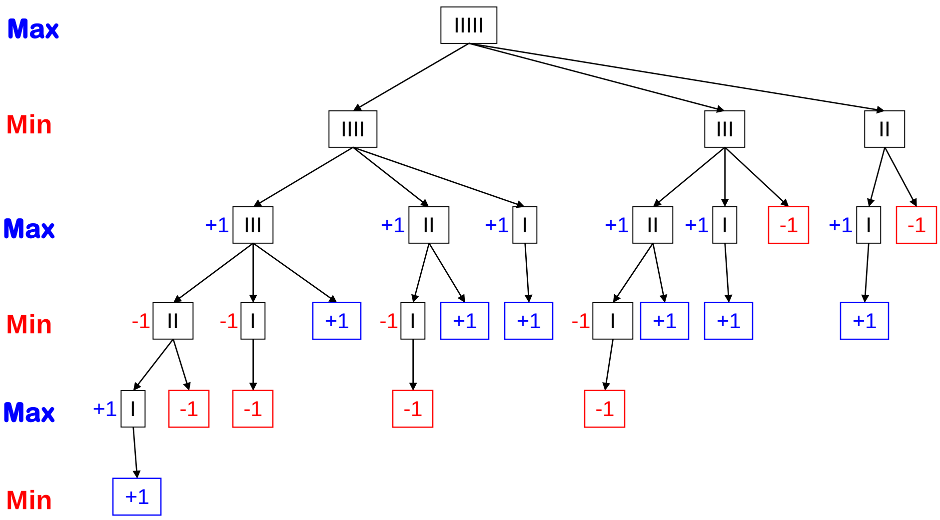
Minimax Values in Nim Game Tree



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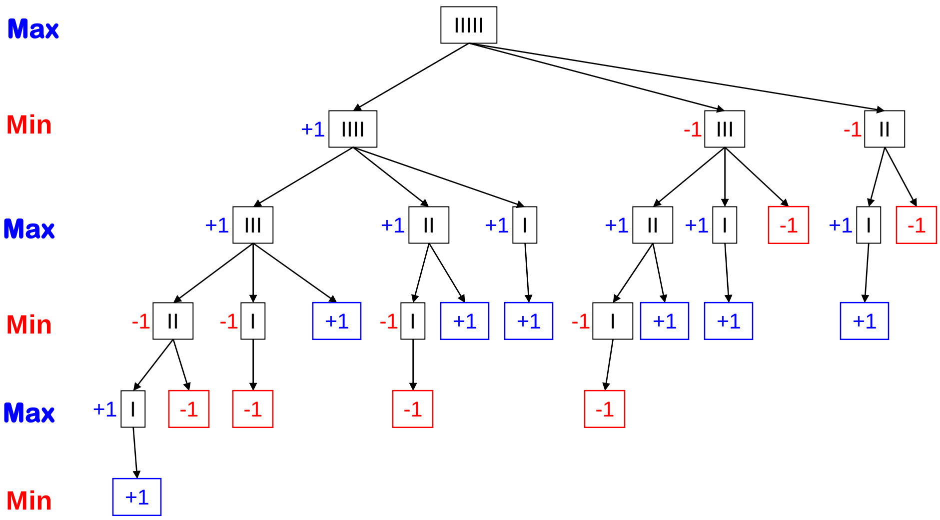
Minimax Values in Nim Game Tree



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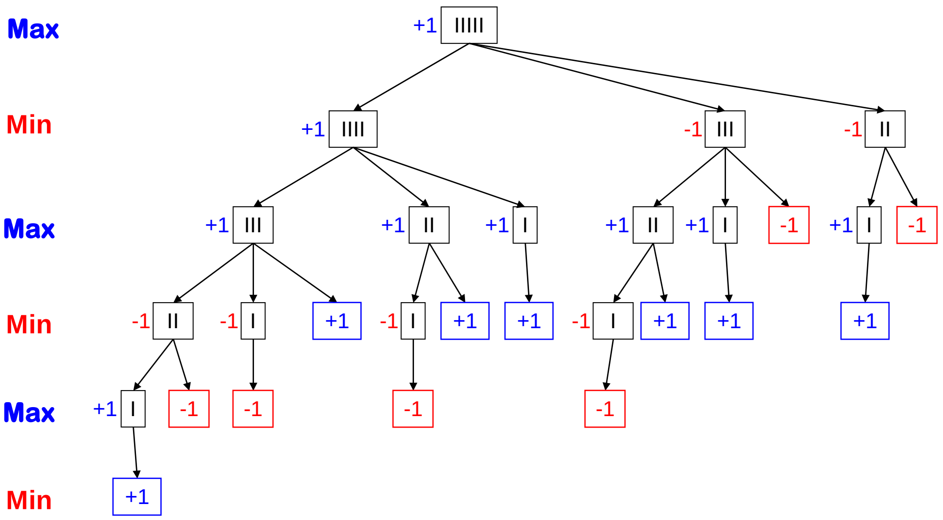
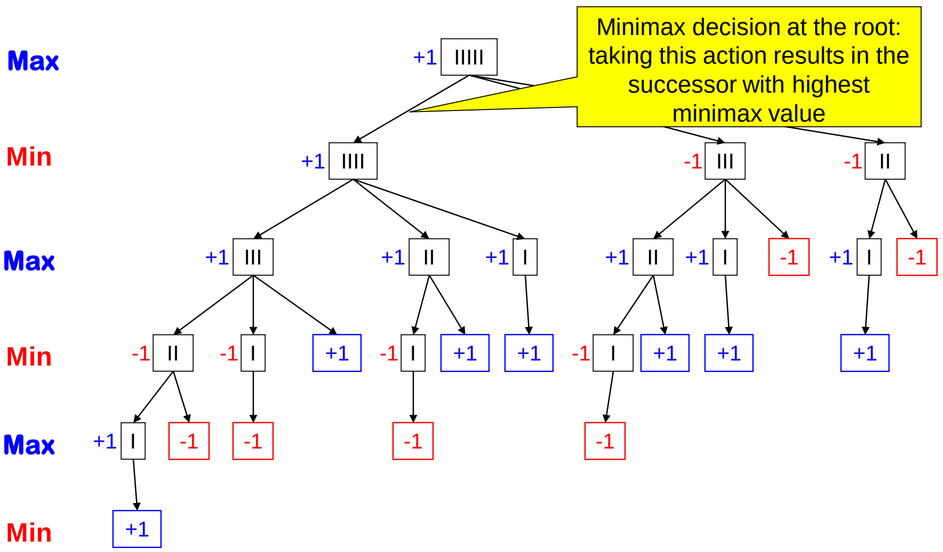
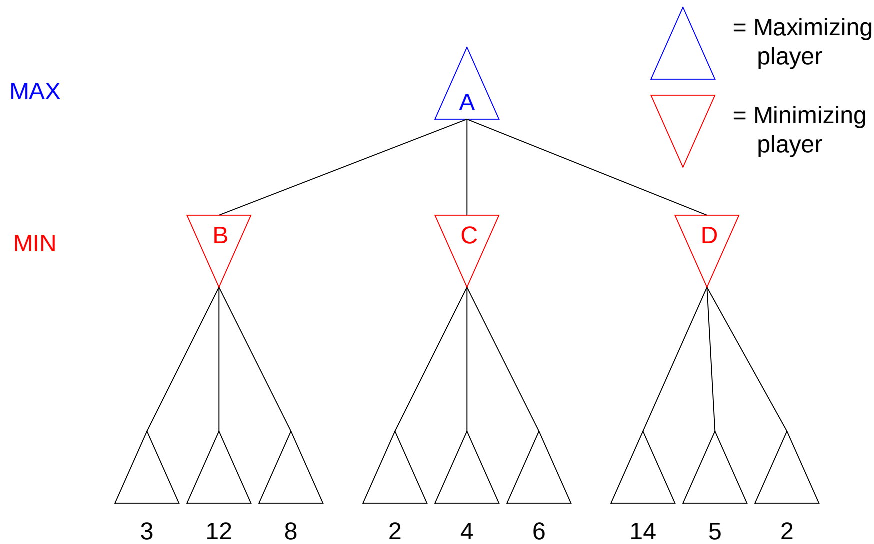
Minimax Values in Nim Game Tree



Minimax Values in Nim Game Tree



Another Example



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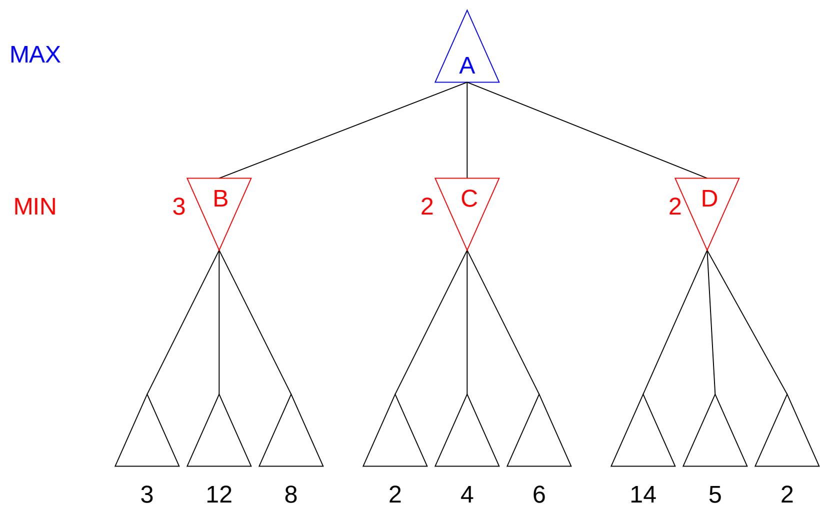

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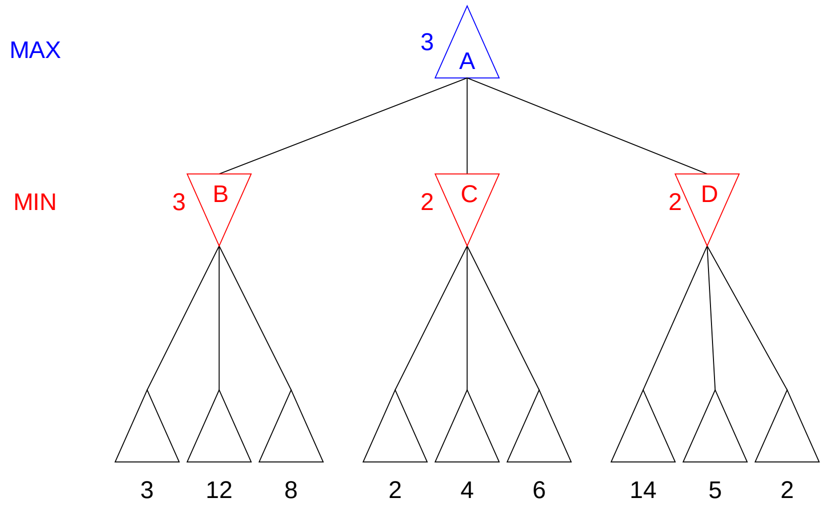

The MINIMAX Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
    inputs: *state*, current state in game
    v ← MAX-VALUE(*state*)
    return the *action* in SUCCESSORS(*state*) with value *v*

function MAX-VALUE(*state*) **returns** *a utility value*
    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    v ← - Infinity
    for *a*, *s* in SUCCESSORS(*state*) **do**
        v ← MAX(*v*, MIN-VALUE(*s*))
    return *v*

function MIN-VALUE(*state*) **returns** *a utility value*
    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    v ← Infinity
    for *a*, *s* in SUCCESSORS(*state*) do
        v ← MIN(*v*, MAX-VALUE(*s*))
    return *v*

The MINIMAX algorithm

- Computes minimax decision from the current state
- Depth-first exploration of the game tree
- Time Complexity $O(b^m)$ where b=# of legal moves, m=maximum depth of tree
- Space Complexity:
  - O(bm) if all successors generated at once
  - O(m) if only one successor generated at a time (each partially expanded node remembers which successor to generate next)

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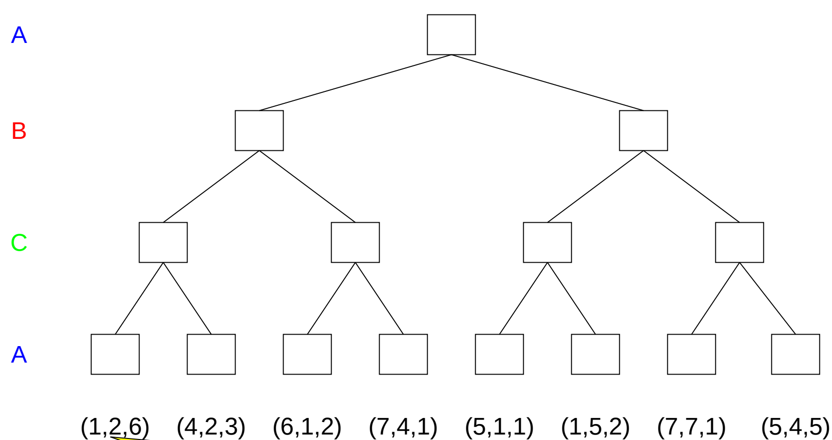
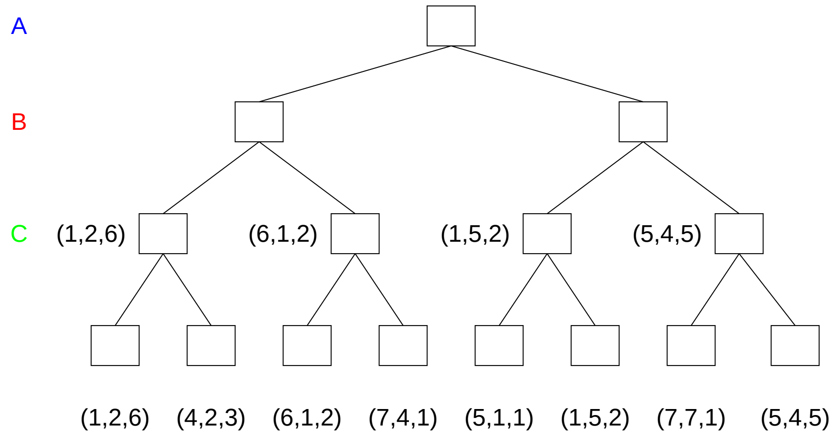
Minimax With 3 Players



Now have a vector of utilities for players (A,B,C). All players maximize their utilities. Note: In two-player, zero-sum games, we have a single value because the values are always opposite.

Minimax With 3 Players



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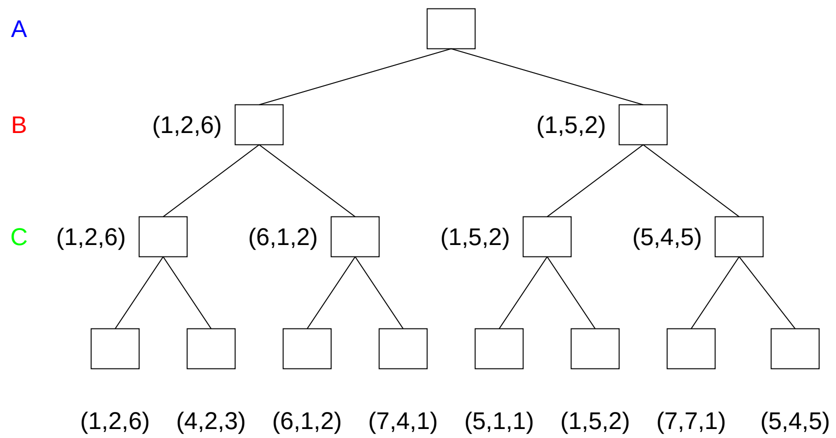
Minimax With 3 Players



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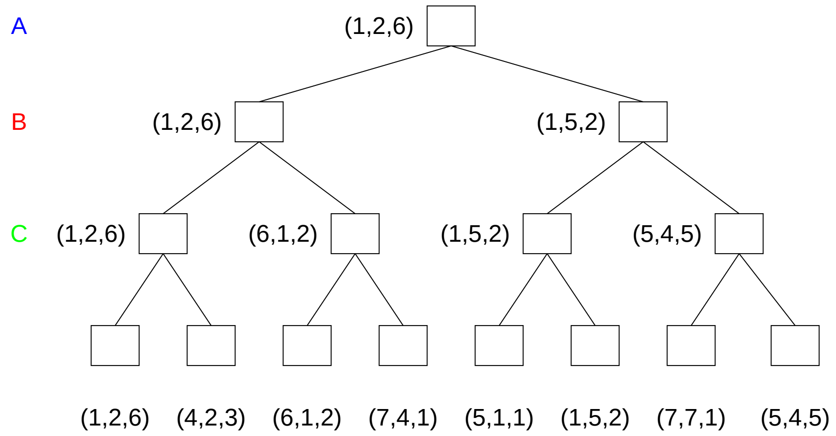
Minimax With 3 Players



Subtleties With Multiplayer Games

- Alliances can be made and broken
- For example, if A and B are weaker than C, they can gang up on C
- But A and B can turn on each other once C is weakened
- But society considers the player that breaks the alliance to be dishonorable

Pruning

- Can we improve on the time complexity of $O(b^m)$?
- Yes if we prune away branches that cannot possibly influence the final decision

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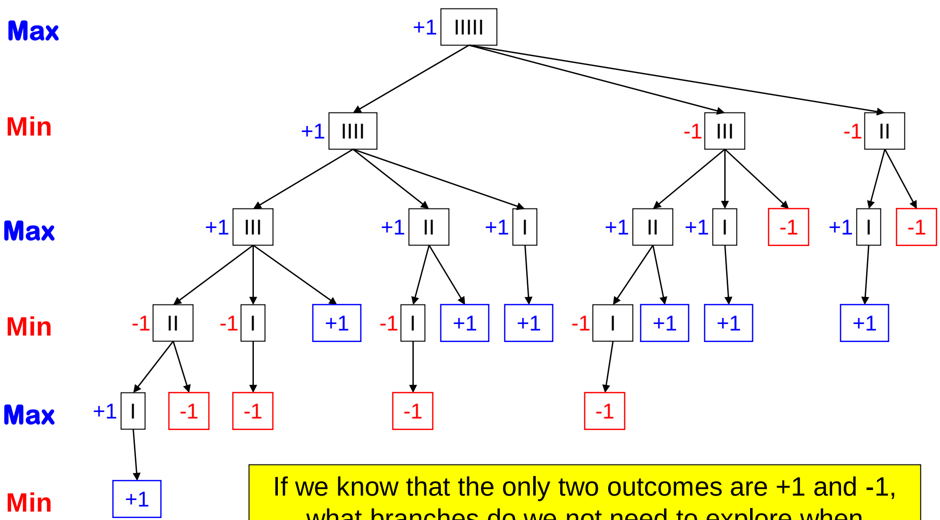
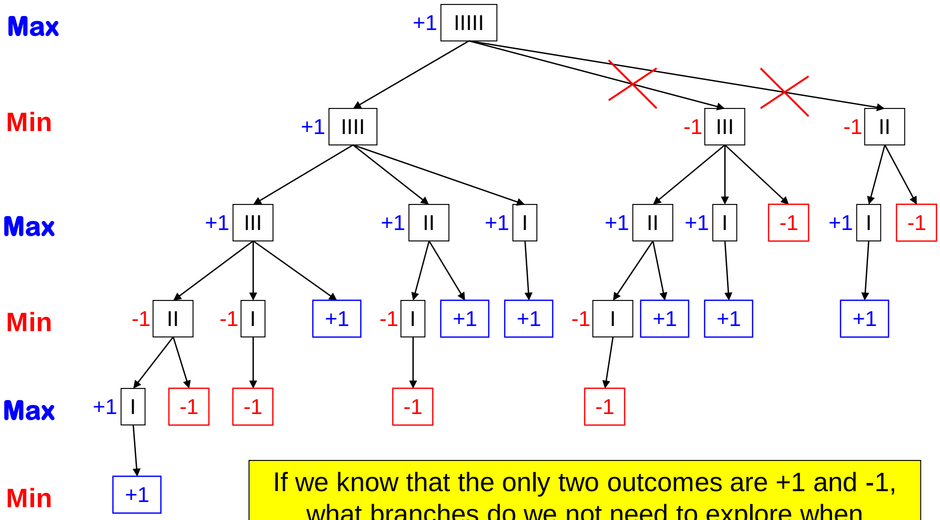
Pruning in Nim



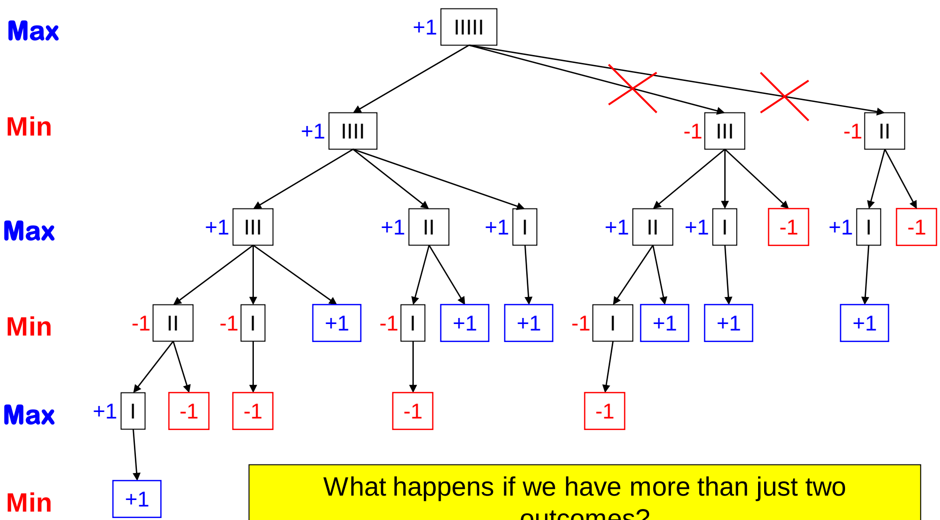
If we know that the only two outcomes are +1 and -1, what branches do we not need to explore when minimax backtracks?

Pruning in Nim



If we know that the only two outcomes are +1 and -1, what branches do we not need to explore when minimax backtracks?

Pruning in Nim



What happens if we have more than just two outcomes?

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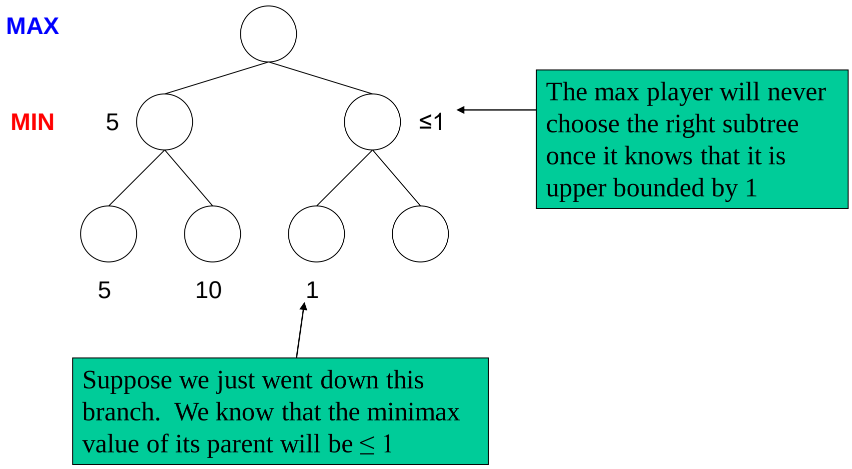
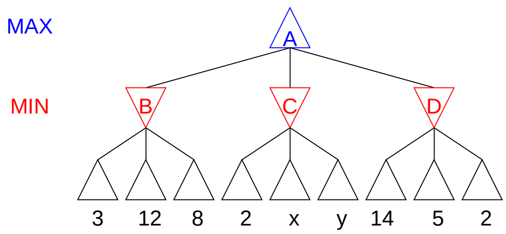
Pruning Intuition (General Case)



Pruning Example



MINIMAX-VALUE(root)

= max(min(3,12,8),min(2,x,y),min(14,5,2))

= max(3,min(2,x,y),2)

= max(3,z,2) where z ≤ 2

= 3

Pruning Intuition

Remember that minimax search is DFS.

At any one time, we only have to consider the nodes along a single path in the tree

In general, let:

- α = highest minimax value of all of the MAX player's choices expanded on current path
- β = lowest minimax value of all of the MIN player's choices expanded on current path
- If at a MIN player node, prune if minimax value of node $\leq \alpha$
- If at a MAX player node, prune if minimax value of node $\geq \beta$

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ALPHA-BETA Pseudocode

function ALPHA-BETA-SEARCH(*state*) **returns** an action
    inputs: *state*, current state in game
    v ← MAX-VALUE(*state*, -∞, +∞)
    return the *action* in SUCCESSORS(*state*) with value *v*

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
    inputs: *state*, current state in game
        α, the value of the best alternative for MAX along the path to *state*
        β, the value of the best alternative for MIN along the path to *state*

    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    v ← -∞
    for *a*, *s* in SUCCESSORS(*state*) **do**
        v ← MAX(*v*, MIN-VALUE(s, α, β))
        if *v* ≥ β **then return** *v*
        α ← MAX(α, v)
    return v

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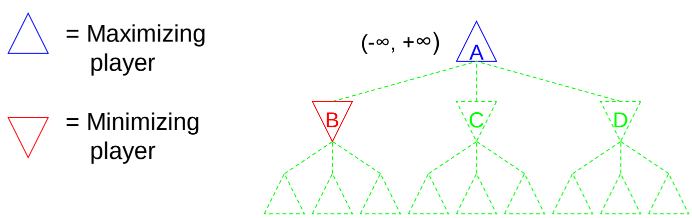
ALPHA-BETA Pseudocode

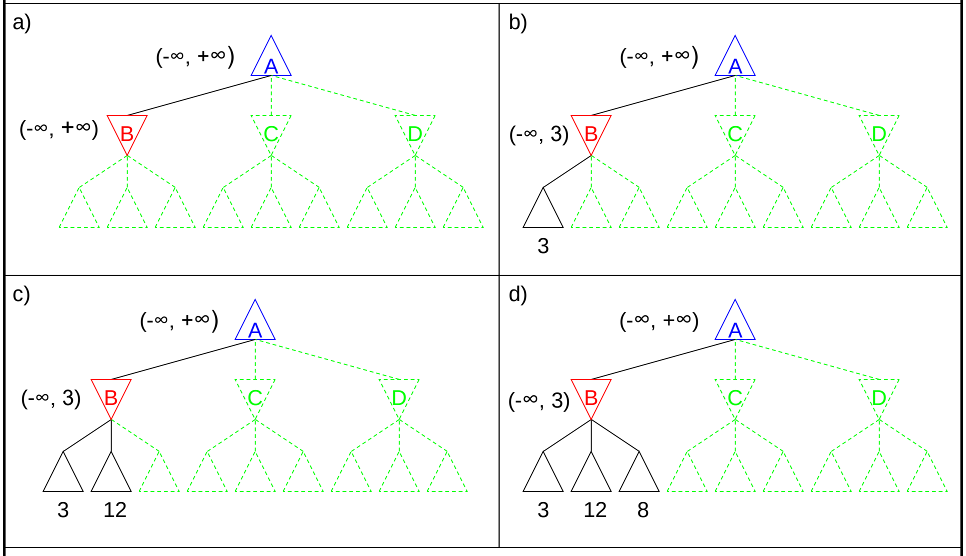
function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  v ← +∞
  for *a*, *s* in SUCCESSORS(*state*) do
    v ← MIN(*v*, MAX-VALUE(s, α, β))
    if *v* ≤ α **then return** *v*
    β ← MIN(β, *v*)
  return *v*

Illustrating the Pseudocode

- In the example to follow, the notation (-∞, +∞) represents the (α, β) values for the corresponding node
- This example is intended to illustrate how the actual implementation of Alpha-Beta pruning works



Alpha-Beta Pruning Example



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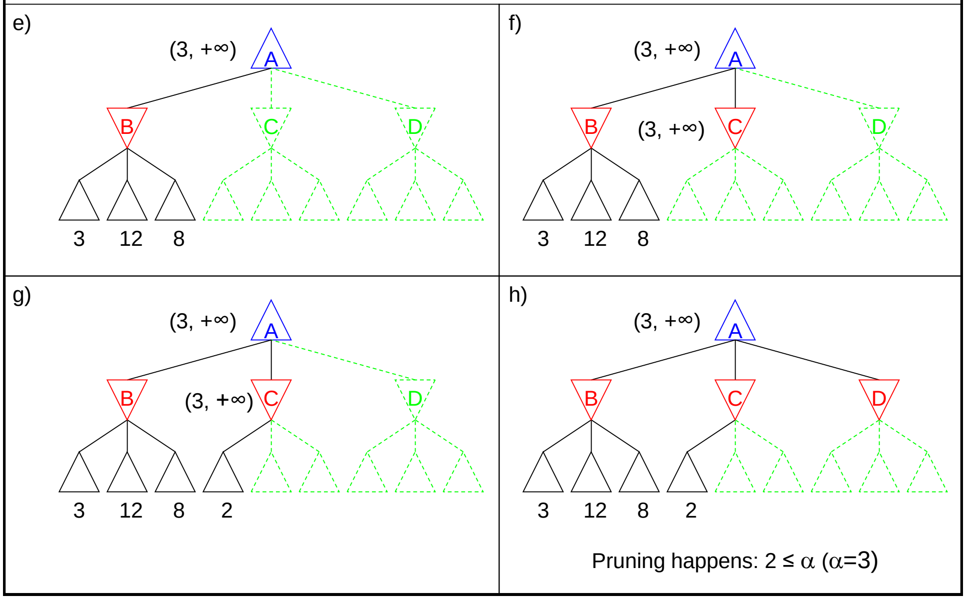
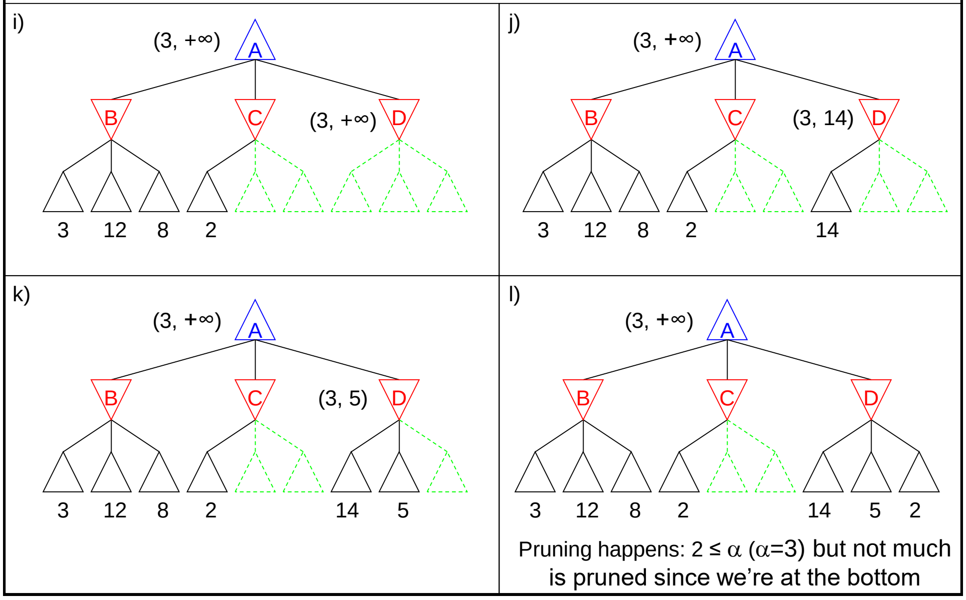


Alpha-Beta Pruning Example



Effectiveness of Alpha-Beta

- Depends on order of successors
- Best case: Alpha-Beta reduces complexity from $O(b^m)$ for minimax to $O(b^{m/2})$
- This means Alpha-Beta can lookahead about twice as far as minimax in the same amount of time

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Implementation Details

- In games we have the problem of transposition
- Transposition means different permutations of the move sequence that end up in the same position
- Results in lots of repeated states
- Use a transposition table to remember the states you've seen (similar to closed list)

What you should know

- Be able to draw up a game tree
- Know how the Minimax algorithm works
- Know how the Alpha-Beta algorithm works
- Be able to do both algorithms by hand