

PROPOSITIONAL LOGIC 4/25/18

$\alpha \models \beta$ iff $\alpha \Rightarrow \beta$ is valid
" $\neg\alpha \vee \beta$ "
 $\neg(\neg\alpha \vee \beta)$ is unsat.
 $\alpha \wedge \neg\beta$ "

$(A \Rightarrow B) \vee (\neg A \Rightarrow \neg B)$
 $\neg A \vee B$ \vee $A \vee \neg B$ \rightarrow valid

1. $0 \Rightarrow \neg G$
2. $(0 \wedge S) \Rightarrow C$
3. $(\neg R \vee G) \Rightarrow B$
4. $S \wedge 0$
5. $C \Rightarrow \neg R$

KB \models B?

2+4: $\frac{(S \wedge 0), (0 \wedge S) \Rightarrow C}{C}$

6. C

5+6: $\frac{C, C \Rightarrow \neg R}{\neg R}$

7. $\neg R$

3+7: $\frac{\neg R, (\neg R \vee G) \Rightarrow B}{B}$

B ✓

KB: 1. $P \Rightarrow M \rightarrow \neg P \vee M$
2. $S \Rightarrow P \rightarrow \neg S \vee P$

KB $\models S \Rightarrow M$?

$\hookrightarrow \neg S \vee M$

$\frac{\neg P \vee M, \neg S \vee P}{\neg S \vee M}$

$\neg S \vee M$

$(B \vee C) \Rightarrow D$

$\neg(B \vee C) \vee D$

$(\neg B \wedge \neg C) \vee D$

$(\neg B \vee D) \wedge (\neg C \vee D)$

5. R

5+3: 6. $\neg E$

6+1: 7. $\neg C$

6+2: 8. $\neg P$

6+4: 9. $\neg A \vee \neg P$

Alternative proof for Socrates example:

KB: 1. $P \Rightarrow M$
2. $S \Rightarrow P$

Does $KB \models S \Rightarrow M$?

| S | P | M | ① $P \Rightarrow M$ | ② $S \Rightarrow P$ | ① \wedge ② KB | query $S \Rightarrow M$ | KB \Rightarrow ($S \Rightarrow M$) |
|---|---|---|------------------------|------------------------|--------------------|----------------------------|--|
| F | F | F | T | T | T | T | T |
| F | F | T | T | T | T | T | T |
| F | T | F | F | T | F | T | T |
| F | T | T | T | T | T | T | T |
| T | F | F | T | F | F | F | T |
| T | F | T | T | F | F | T | T |
| T | T | F | F | T | F | F | T |
| T | T | T | T | T | T | T | T |

Valid.
 $\therefore KB \models S \Rightarrow M.$