

Coin	Card	Candy	P(Coin, Card, Candy)	
tails	black	1	0.15	
tails	black	2	0.06	The probabilities
tails	black	3	0.09	in the last column
tails	red	1	0.02	sum to 1
tails	red	2	0.06	
tails	red	3	0.12	
heads	black	1	0.075	
heads	black	2	0.03	
heads	black	3	0.045	
heads	red	1	0.035	
heads	red	2	0.105	
heads	red	3	0.21	
This	cell m	eans P(Cc	oin=heads, Card=red, Candy	y=3) = 0.21 2
			;	

# Joint Probability Distribution

From the full joint probability distribution, we can calculate any probability involving these three random variables.

e.g. P(*Coin* = *heads* OR *Card* = *red*)

#### Joint Probability Distribution

P(Coin = heads OR Card = red) =

P(Coin=heads, Card=black, Candy=1) + P(Coin=heads, Card=black, Candy=2) + P(Coin=heads, Card=black, Candy=3) + P(Coin=tails, Card=red, Candy=1) + P(Coin=tails, Card=red, Candy=2) + P(Coin=heads, Card=red, Candy=3) + P(Coin=heads, Card=red, Candy=2) + P(Coin=heads, Card=red, Candy=3) = 0.075 + 0.03 + 0.045 + 0.02 + 0.06 + 0.12 + 0.035 + 0.105 +0.21 = 0.7

## Marginalization

We can even calculate marginal probabilities (the probability distribution over a subset of the variables) e.g.:

P(Coin=tails, Card=red) = P(Coin=tails, Card=red, Candy=1) + P(Coin=tails, Card=red, Candy=2) + P(Coin=tails, Card=red, Candy=3) = 0.02 + 0.06 + 0.12 = 0.2

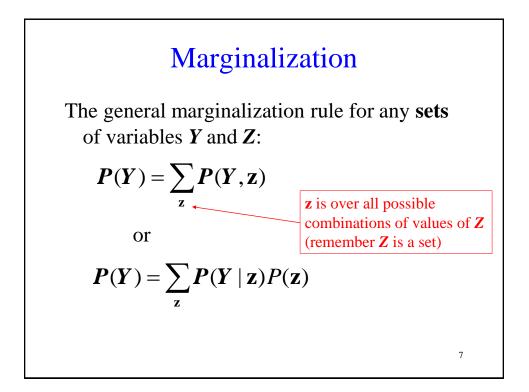
#### Marginalization

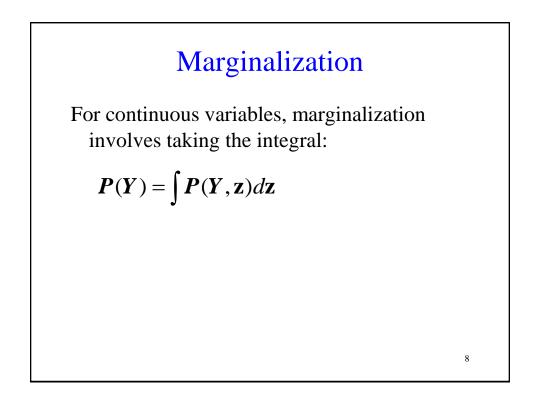
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Or even:

P(Card=black) = P(Coin=heads, Card=black, Candy=1) + P(Coin=heads, Card=black, Candy=2) + P(Coin=heads, Card=black, Candy=3) + P(Coin=tails, Card=black, Candy=1) + P(Coin=tails, Card=black, Candy=2) + P(Coin=tails, Card=black, Candy=3) = 0.075 + 0.03 + 0.045 + 0.015 + 0.06 + 0.09 = 0.315





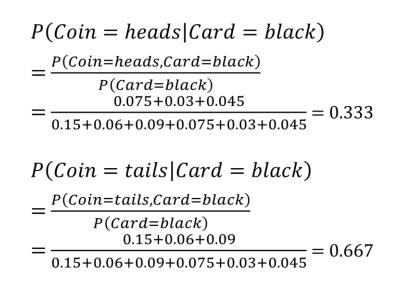
	Pr	act	ice	
	Coin	Card	Candy	P(Coin, Card, Candy)
Compute	tails	black	1	0.15
Compute $P(Candy = 2).$	tails	black	2	0.06
P(Canay = 2).	tails	black	3	0.09
	tails	red	1	0.02
	tails	red	2	0.06
	tails	red	3	0.12
	heads	black	1	0.075
	heads	black	2	0.03
	heads	black	3	0.045
	heads	red	1	0.035
	heads	red	2	0.105
	heads	red	3	0.21
				9

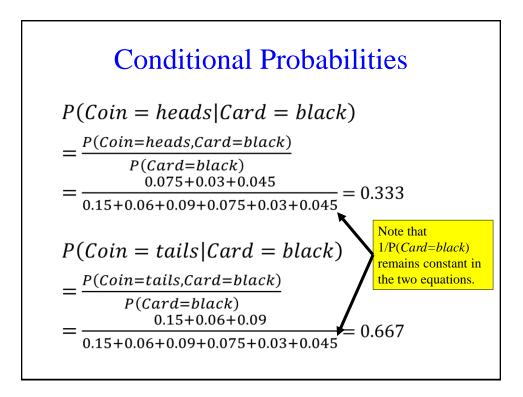
# **Conditional Probabilities**

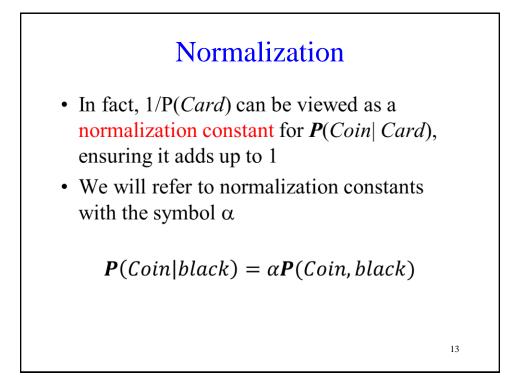
We can also compute conditional probabilities from the joint. Recall:

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

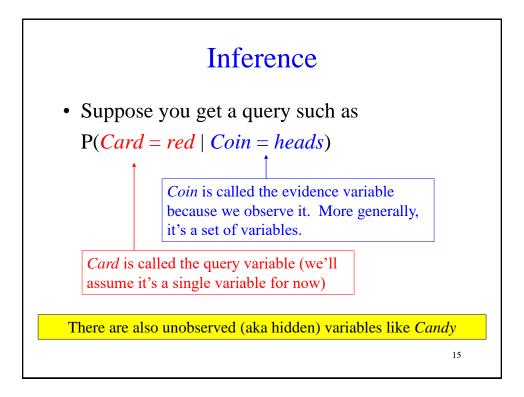
#### **Conditional Probabilities**

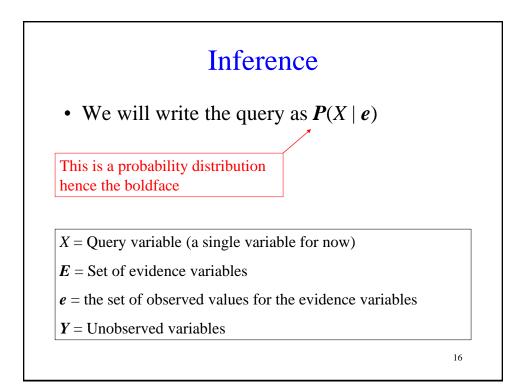


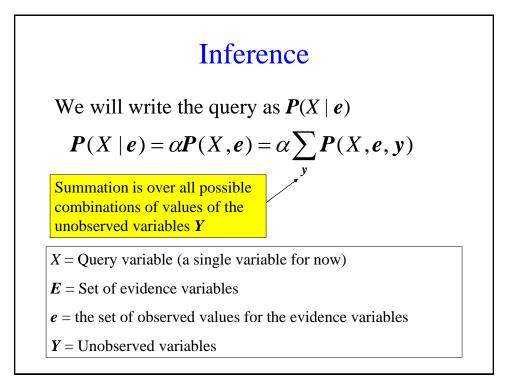


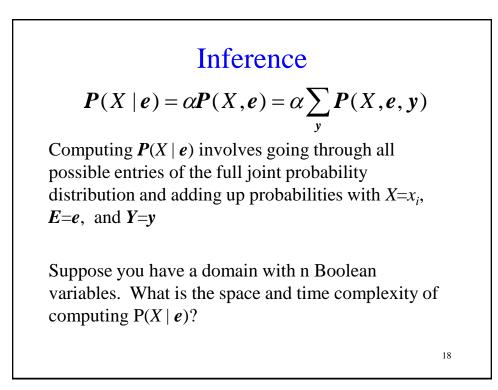


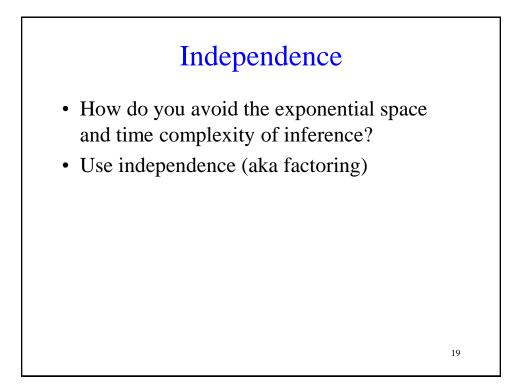
Pr	acti	ce		
	Coin	Card	Candy	P(Coin, Card, Candy)
Compute	tails	black	1	0.15
P(Candy = 1 Card = red).	tails	black	2	0.06
	tails	black	3	0.09
	tails	red	1	0.02
	tails	red	2	0.06
	tails	red	3	0.12
	heads	black	1	0.075
	heads	black	2	0.03
	heads	black	3	0.045
	heads	red	1	0.035
	heads	red	2	0.105
	heads	red	3	0.21
				14

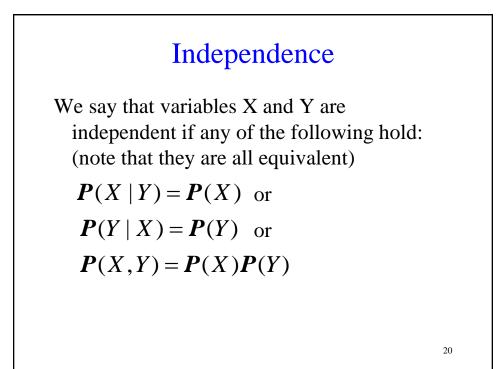


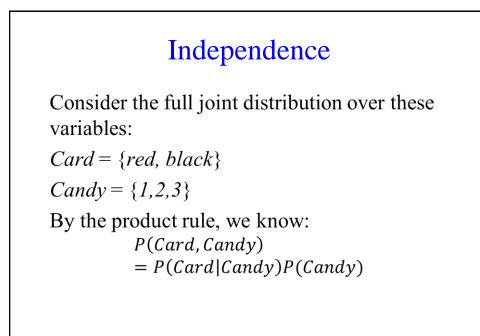


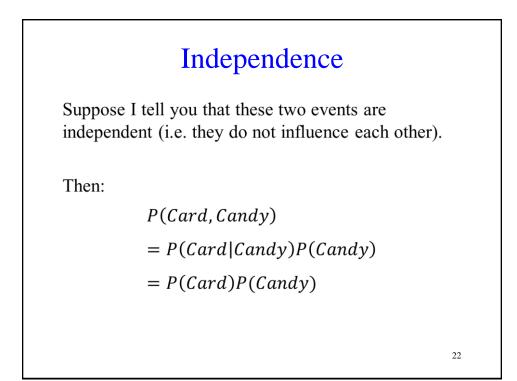


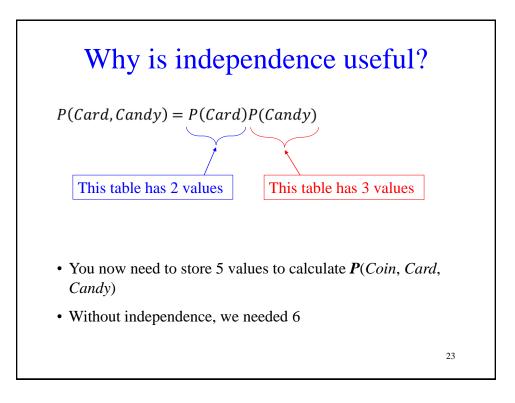


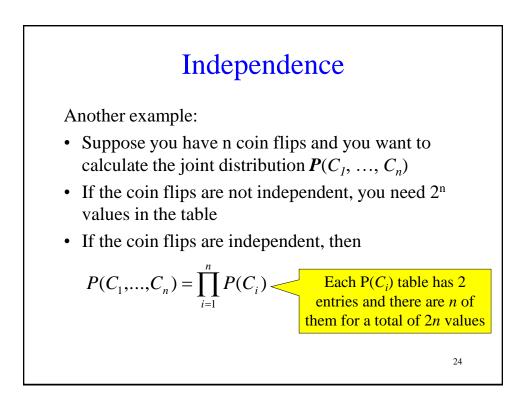












## Independence

- Independence is powerful!
- It required extra domain knowledge. A different kind of knowledge than numerical probabilities. It needed an understanding of relationships among the random variables.

P	Practi	ce		
	Coin	Card	Candy	P(Coin, Card, Candy)
Are Coin and Card	tails	black	1	0.15
index and ant in this	tails	black	2	0.06
independent in this	tails	black	3	0.09
distribution?	tails	red	1	0.02
	tails	red	2	0.06
	tails	red	3	0.12
Recall:	heads	black	1	0.075
$\boldsymbol{P}(X \mid Y) = \boldsymbol{P}(X)$	heads	black	2	0.03
	heads	black	3	0.045
$\boldsymbol{P}(Y \mid X) = \boldsymbol{P}(Y)$	heads	red	1	0.035
$\mathbf{D}(\mathbf{V}   \mathbf{V}) = \mathbf{D}(\mathbf{V}) \mathbf{D}(\mathbf{V})$	heads	red	2	0.105
$\boldsymbol{P}(X,Y) = \boldsymbol{P}(X)\boldsymbol{P}(Y)$	) heads	red	3	0.21

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