ECE 353 Probability and Random Signals - Homework 3

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Instructor: Dr. Raviv Raich
School of Electrical Engineering and Computer Science
Oregon State Univeristy

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Q1. An ID includes 10 digits selected uniformly random in range from integral number 0-9. Each number can be selected multiple times. What is the probability that none of the digits is 0, 3,7 if the first digit is not 0?

Solution 1

Define two events firstly:

- A = none of the digit digits is 0, 3,7
- B = the first digit is not 0

Then, the probability that none of the digit can be 0 or 3 or 7 if the first digit is not 0 can be calculated as:

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} = \frac{7^{10}/10^{10}}{9 * 10^9/10^{10}} = \frac{7^{10}}{9 * 10^9} \tag{1}$$

Q2. Suppose a language containing five letters: A, B, C, D, E, F.

(a) How many three-letter words can you form in this language?

(b) How many four-letter words can you form if each letter appears only once in each word?

(c) What is the probability that a three-letter word (with each letter appearing only once) contains E?

Solution 2

(a) Since each letter can take on any one of the 6 possible letters in the alphabet, the number of 3 letter words that can be formed is $6^3 = 216$.

(b) If we allow each letter to appear only once then we have 6 choices for the first letter and 5 choices for the second, 4 choices for the third letter and 3 choices for the last. Therefore, there are a total of $6 \times 5 \times 4 \times 3 = 360$ possible words.

(c) The denominator is the answer to (b) and the numerator should be $\binom{5}{2} \times 3 \times 2 \times 1$ where the first term is choosing 2 letters from the rest 5 and the rest terms denote the permutation of all the three letters (with E). So the result would be $\frac{\binom{5}{2} \times 3 \times 2 \times 1}{6 \times 5 \times 4} = \frac{1}{2}$

Q3. Consider a binary code with 6 bits (0 or 1) in each code word. An example of a code word is 010101. In each code word, a bit is a zero with probability 0.7, independent of any other bit.

(a) What is the probability of the code word 000111?

(b) What is the probability that a codeword contains exactly three ones?

Solution 3

(a) Since the probability of a zero is 0.8, we can express the probability of the code word 000111 as 2 occurrences of a 0 and three occurrences of a 1. Therefore

$$P[000111] = (0.7)^3(0.3)^3 = 0.0093 \tag{2}$$

(b) The probability that a code word has exactly three 1's is

$$P[\text{three 1's}] = \binom{6}{3}(0.7)^3(0.3)^3 = 0.1852 \tag{3}$$

Q4. In a poker-like dice game, five fair 6-sided dice are rolled independently. Consider the following two events: 'four of a kind' - all but one of the dice show the same number (e.g., 46444) and 'a 6-high straight flush' - the five dice outcome contains exactly one of each: 2, 3, 4, 5,and 6 (e.g., 45236)

(a) Determine the number of possible outcomes in this five dice roll.

(b) Determine the probability of 'four of a kind'.

(c) Determine the probability of 'a 6-high straight flush'.

Solution 4

(a) For five dice roll, the number of possible outcomes is: 6^5

(b) For 'four of a kind', the number of outcomes is: $6 * 5 * 5 = 150$

$$P = \frac{150}{6^5} = \frac{150}{7776} \tag{4}$$

(c) For the 'a 6-high straight flush', the number of outcomes is $5! = 120$

$$P = \frac{120}{6^5} = \frac{120}{7776} \tag{5}$$