ECE 353 : Probability and Random Signals
Homework 2
Spring 2019

Due April 16 2019

1. Consider the binary communication channel shown in the following figure. The channel input symbol

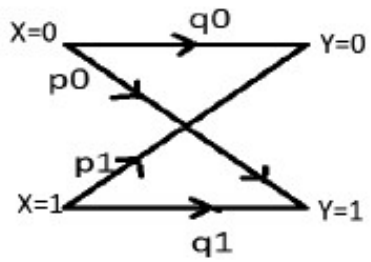

Figure 1: A binary communication channel

X may assume the state 0 or 1. Similarly, the channel output symbol may assume either the state 0 or 1. Because of the channel noise, an input 0 may convert to an output 1 and vice versa. The channel is characterized by the channel transition probabilities $p_0; q_0; p_1; q_1$, denoted by

$$p_0 = P(y_1|x_0), \quad p_1 = P(y_0|x_1) \tag{1}$$
$$q_0 = P(y_0|x_0), \quad q_1 = P(y_1|x_1) \tag{2}$$

where x_0, x_1 denote the events $\{X = 0\}$ and $\{X = 1\}$ respectively and y_0, y_1 denote the events $\{Y = 0\}$ and $\{Y = 1\}$ respectively. Note that

$$p_0 + q_0 = 1 = p_1 + q_1.$$

Let $P(x_0) = 0.5;\ p_0 = 0.2;\ p_1 = 0.3.$

   (a) Find $P(y_0)$ and $P(y_1)$.
   (b) If a 0 was observed at the output, what is the probability that a 0 was the input state?
   (c) If a 1 was observed at the output, what is the probability that a 1 was the input state?
   (d) Calculate the probability of error P_e.

2. A factory produces its entire output with 3 machines. Machines I, II and III produce 80%, 10% and 10% of the output, respectively, but 5%, 10% and 15% of their outputs are defective, respectively. What fraction of the total output is defective?

3. Let A, B be events in a sample space S. Show that if A, B are independent, then so are $\bar{A}, \bar{B}$.

4. Consider two urns. The first contains two white and seven black balls, and the second urn contains five white and six black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected?