ECE 353 : Probability and Random Signals
Homework 4
Spring 2019

Due April 30, 2019

1. Random variable Y has a probability mass function (pmf) as

$$p_Y(y) = \begin{cases} \frac{c}{y}, & y = 1, 2 \\ \frac{c}{y^2}, & y = -1, -2 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of the constant c.

   (b) Calculate

      i. $P(Y = -2)$

      ii. $P(Y < 1)$

2. Assume the resistance of R is a random variable, uniformly distributed on the interval $[850\Omega, 1150\Omega]$.

   (a) Find the PDF.

   (b) Calculate $P(900\Omega \leq 950\Omega)$?

3. In a restaurant, the time (in minutes) that a customer has to wait before s/he gets a table is specified by the following CDF:

$$F_X(x) = \begin{cases} \frac{x^2}{2}, & 0 \leq x \leq 1, \\ \frac{1}{2}, & 1 \leq x \leq 8, \\ \frac{x}{4} - \frac{3}{2}, & 8 \leq x \leq 10, \\ 1, & x \geq 10. \end{cases}$$

   (a) Compute and sketch the PDF $f_X(x)$.

   (b) Verify the area under the PDF is indeed unity.

   (c) What is the probability that the customer will have to wait at least 5 minutes?

4. Consider the function given by

$$F(x) = \begin{cases} 0, & x < 0 \\ x + \frac{1}{2}, & 0 \leq x \leq \frac{1}{2} \\ 1, & x \geq \frac{1}{2}. \end{cases}$$

   (a) Sketch $F(x)$ and show that $F(x)$ satisfies the properties of a cdf.

   (b) If X is the random variable whose cdf is given by $F(x)$, find

      i. $P(X \leq 1/4)$,

      ii. $P(0 < X \leq 1/4)$.