ECE 353 : Probability and Random Signals
Homework 8
Spring 2019

Due May 30, 2019

1. Consider two random variables X and Y that follows the joint PDF:

$$f_{XY}(x,y) = \begin{cases} c, & x + y < 5, \ x \geq 0, \ y \geq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

   (a) Find the value of c.

   (b) Prove that X and Y are not independent.

2. Let $\{X(t) : t \geq 0\}$ be a Poisson process *i.e.*, $P(X(t) = n) = \frac{\{\lambda t\}^n exp(-\lambda t)}{n!}$. For $s = t/5$, show that the conditional distribution of $X(s)$ given that $X(t) = n$ is binomial with parameters n and $p = 1/5$, *i.e.*,

$$P(X(t/5) = m|X(t) = n) = \binom{n}{m}(1 - p)^{n-m}p^m.$$

3. The joint PDF of X,Y is as follows.

$$f_{XY}(x,y) = \begin{cases} ce^{-x}e^{-y}, & x \geq 0, y \geq 0. \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

   (a) Find the value of c.

   (b) Find $f_Y(y)$.

   (c) Find $f_{X|Y}(x|y)$.