ECE 353 Probability and Random Signals - Homework 9

Spring 2019
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Due: Jun. 6, 2019

Q1. For a random variable X, let $Y = aX + b$. Show that if $a > 0$ then $\rho_{X,Y} = 1$. Also show that if $a < 0$, then $\rho_{X,Y} = -1$.

Q2. Let the random variable X which can be represented as a sum of random variables $X = \sum_{i=1}^{n} X_i$. Show that, if X_i and Y_i are uncorrelated, i.e. $\text{Cov}(X_i, Y_i) = 0$ or $E[X_i X_j] = E[X_i]E[X_j]$ for every pair of i and j with $1 \le i < j \le n$, then $\text{Var}[X] = \sum_{i=1}^{n} \text{Var}[X_i]$.

Q3. Random variables X and Y have joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} cxy^2 & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the constant c.

(b) Find $P[X > Y]$ and $P[Y < X^2]$.

(c) Find $P[\min(X, Y) \le 1/2]$.

Q4. Let$X$ and Y be two independent normal random variables $N(0, 1)$. Show that $aX + bY$ and $bX - aY$ are independent random variables.

Q5. Consider the two independent RVs $X \sim U[-1, 1]$ and $Y \sim U[-1, 1]$. Let $Z = X^2 Y$.

(a) Find the mean of Z, $E[Z]$.

(b) Find $\text{Corr}(X, Z)$ and $\text{Corr}(Y, Z)$.

(c) Determine if Z and Y are uncorrelated.

Q6. You are interested in estimating the probability of A, denoted as $P[A]$, where

$$A = \{\text{a student in EECS at OSU wants to go to graduate school}\}.$$

You start asking your fellow students "do you want to go to graduate school?" so that you can make an estimation. Suppose every fellow student who you've asked answers your question with 'yes' or 'no'. Assume that the students' answers are independent and identically distributed. Denote $\hat{P}_n(A) = \frac{\text{number of 'yes'}}{n}$ when you have answers of n fellow students. If you wish to make

$$|\hat{P}_n(A) - P[A]| < 0.1$$

with a confidence level 0.95, how many fellow students do you need to ask, in the worst case?