

$$C = \frac{a}{a^2}$$

$$f_X(x) = \begin{cases} 0 \\ \dots \end{cases}$$

$$b) E[X] = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^a x \cdot \frac{2}{a^2} x dx$$

$$= \frac{2}{a^2} \int_0^a x^2 dx = \frac{2}{a^2} \left[\frac{x^3}{3} \right]_0^a = \frac{2}{a^2} \cdot \frac{a^3}{3} = \frac{2a}{3}$$

$$c) E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x) dx = \int_0^a x^2 \cdot \frac{2}{a^2} x dx = \frac{2}{a^2} \int_0^a x^3 dx$$

$$= \frac{2}{a^2} \left[\frac{x^4}{4} \right]_0^a = \frac{2a^4}{4} = \frac{a^4}{2}$$

$$\left[\frac{a^2}{2} \right]$$

$$d) \text{Var}(X) = E[X^2] - (E[X])^2 = \frac{a^4}{2} - \left(\frac{2a}{3} \right)^2 = \frac{a^4}{2} - \frac{4a^2}{9}$$

$$b) E[X^2] = \sum_{x \in X} x^2 P_X(x) = \sum_{x=0}^1 x^2 P_X(x) =$$

$$c) \text{Var}(X) = E[X^2] - (E[X])^2 = P - P^2 = \boxed{P}$$

$$= \int_0^{\infty} \lambda e^{-\lambda y^2} |2y|, y \geq 0$$

$$= \int_0^{\infty} 0, \text{ o.w.}$$

$$= \int_0^{\infty} 2\lambda y e^{-\lambda y^2}, y \geq 0$$

$$= \int_0^{\infty} 0, \text{ o.w.}$$

⑥

$$E[Y^2] = e^{\text{easy way}}$$

$$E[X] = \frac{1}{\lambda}$$

(From

Derivation:

$$= \int_0^{\infty} x \lambda e^{-\lambda x} dx$$

$$= \int_0^{\infty} x = \frac{t}{\lambda} dt$$

$$\textcircled{b} \quad E[(Y - E(Y))] = ?$$

$$E(Y) = a \underbrace{E(X)}_{\substack{\text{from a)}}} = 0.$$

$$\Rightarrow E[(Y - E(Y))] = E[(Y - 0)] = E(Y)$$

Note also: $E(Y) = 0$, the answer to (a) and the same / moment = centered moment if $E(Y) = 0$

$$f_Y(y) = \begin{cases} f_X(g_1^{-1}(y)) \left| \frac{dg_1^{-1}(y)}{dy} \right| + f_X(g_2^{-1}(y)) \left| \frac{dg_2^{-1}(y)}{dy} \right| & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$$f_X(x) = \begin{cases} \frac{1}{2} & -1 \leq x \leq 1 \\ 0 & \text{o.w.} \end{cases}$$

$$y \geq 0 \\ g_1^{-1}(y) = -\sqrt{y} \\ g_2^{-1}(y) = \sqrt{y}$$

$$\begin{aligned} \rightarrow \frac{dg_1^{-1}(y)}{dy} &= -\frac{1}{2\sqrt{y}} & \rightarrow \left| \frac{dg_1^{-1}(y)}{dy} \right| &= \frac{1}{2\sqrt{y}} \\ \rightarrow \frac{dg_2^{-1}(y)}{dy} &= +\frac{1}{2\sqrt{y}} & \rightarrow \left| \frac{dg_2^{-1}(y)}{dy} \right| &= \frac{1}{2\sqrt{y}} \end{aligned}$$

$$\Rightarrow f_Y(y) = \begin{cases} \frac{1}{2} \cdot \frac{1}{2\sqrt{y}} + \frac{1}{2} \cdot \frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}} & -\sqrt{y} \leq 1 \rightarrow \text{o.w.} \\ 0, & \end{cases}$$

$$= \int_{-1}^1 x^2 \frac{1}{2} dx$$

$$= \frac{x^3}{2.3} \Big|_{-1}^1$$

$$= \frac{x^3}{6} \Big|_{-1}^1$$

$x \in X$

$$= \frac{e^5 + e^{-5}}{2}$$

$$\textcircled{6} \quad E[X^n] = \sum_x x^n P_X(x) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \dots$$

$$= \frac{1}{2} (1 + (-1)^n)$$

$$= \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

\textcircled{c}

PMF of X^2

Let $Y = X^2$

PMF of Y is $P_Y(y) =$

Since $x = \pm 1 \rightarrow y = x^2 = 1$

$$= \begin{cases} 1 & \text{if } y = 1 \\ 0 & \text{otherwise} \end{cases}$$