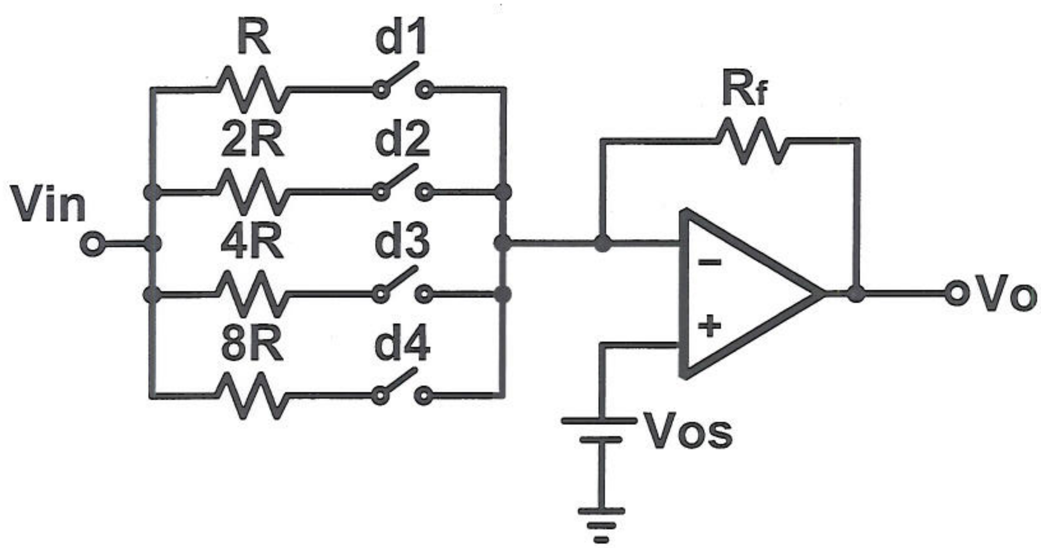
ECE 627

Spring 2012

Final Examination

June 11, 2012, 9:30 – 11:20 am

1. In the multiplying DAC shown, the output signal v_o is proportional to v_{in} and to $|D|$, where $|D|$ is the value of the digital input word, D = $d_1d_2d_3d_4$.



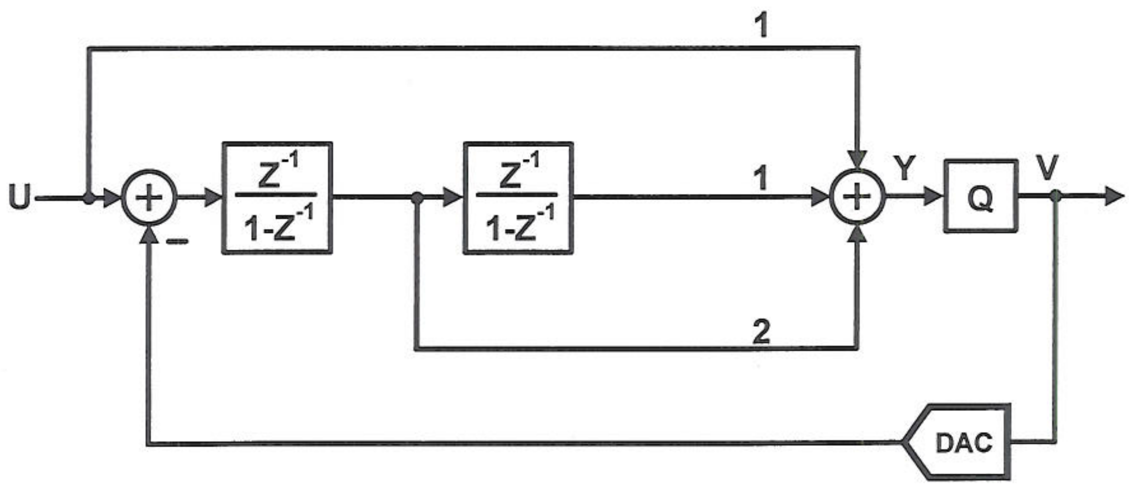
a) Find the value of R_f such that for V_{os}=0 and D=1111, v_o=$-v_{in}$.

b) What is v_o if V_{os}=0 and D=0010 ?

c) How does a nonzero V_{os} affect v_o ?

d) How large can $|V_{os}|$ be for a given v_{in} if the resulting error in v_o should be less than ±1/2 LSB?

2. The modulator shown below has a 9-level quantizer, operating between ground and a reference voltage $V_{ref} = 1.3$ V. What is the largest peak-to-peak value of a sine-wave input voltage u for which the quantizer is not overloaded? Why?



3. The circuit of the ΔΣ modulator MODA shown in Fig. A is redesigned as illustrated in Fig. B, to allow more time for quantization. The integrator transfer function is $I = 1/(z-1)$.

a. What are the NTF and STF for MODA?

b. Given the a_i, $i = 1, 2, \ldots, N$, what should be the values of the parameters $b_0, b_1, \ldots, b_N, g_0$ of MODB if the NTF is to be preserved?

c. What is the STF for MODB?

d. How should the transfer function of the feed-forward path from the input u and the quantizer in MODB be modified to make the STF the same as that of MODA?

Note: It is better to work in terms of I, not z, when solving this problem!

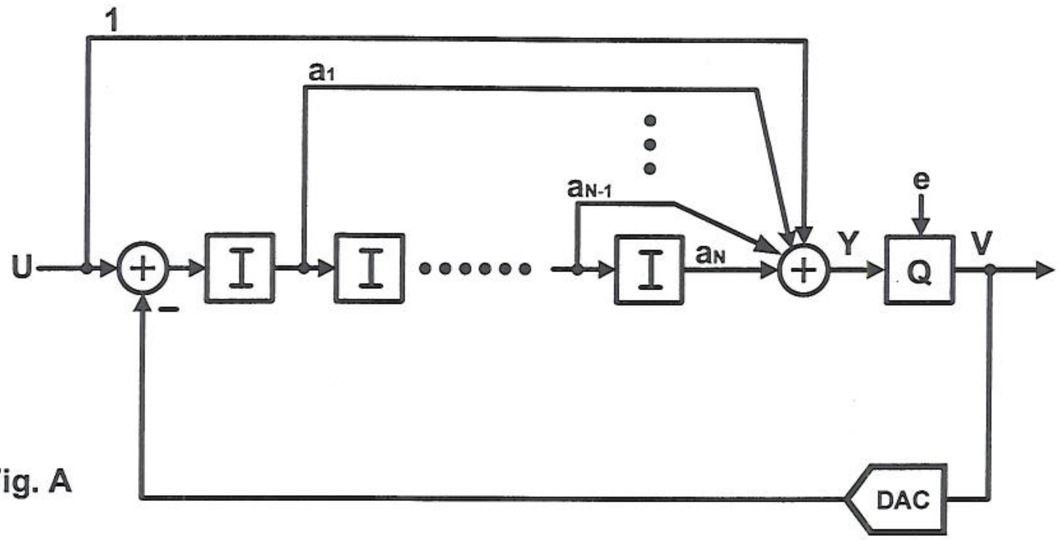


Fig. A

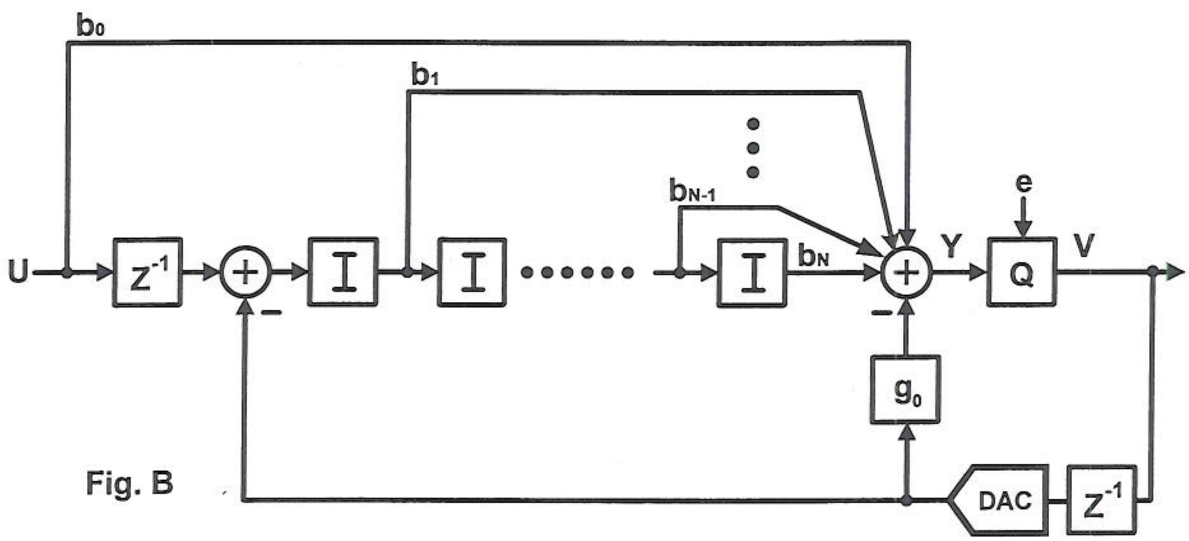


Fig. B

1a. For $D = 1111$, $R_{1111}^{-1} = \frac{1}{R}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right]$

$R_f \stackrel{!}{=} R_{1111} = \frac{8R}{15} \sim 0.533\,R$ ✓ $= \frac{15}{8}\frac{1}{R}$

b. $R_{0010} = 4R$, so

$$V_O = -\frac{R_f}{R_{0010}} = -\frac{2}{15} V_{in} \cong -0.13\bar{3}\, V_{in}$$ ✓

c. From KCL, infinite gain opamp,

$$(V_{in} - V_{OS})/R_{in} = (V_{OS} - V_O)/R_f$$

$$V_{OS}(G_f + G_{in}) = G_{in} V_{in} + G_f V_O$$

$$V_O = \left(1 + \frac{R_f}{R_{in}}\right) V_{OS} - \frac{R_f}{R_{in}} V_{in}$$ ✓

Largest effect for $R_{in} = R_{1111}$:

$$V_O = -V_{in} + 2V_{OS} \,!$$ ✓

d. $V_{LSB} = V_{in} \frac{R_f}{R_{0001}} = V_{in} \frac{R_f}{8R} = \frac{V_{in}}{15}$

$$2|V_{OS}| \stackrel{!}{=} \frac{V_{in}}{30},$$

$$|V_{OS}| \le \frac{V_{in}}{60}$$ ✓

2. $V = E + U + I(2+I)(U-V)$

$$V = U + E/[1+I]^2, \quad I \triangleq \frac{z^{-1}}{1-z^{-1}}$$

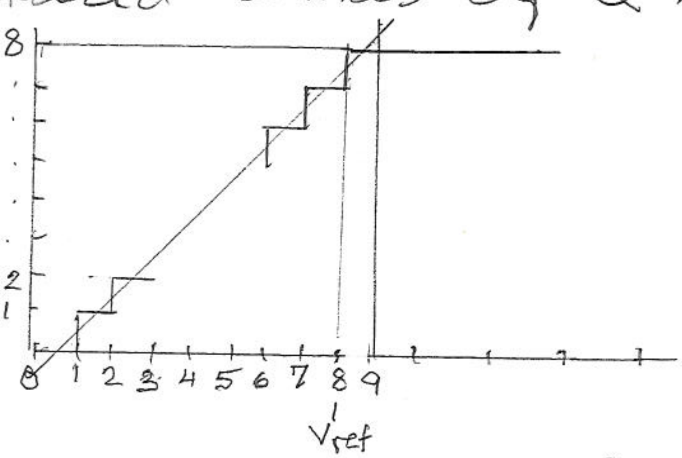
$$1+I = 1/(1-z^{-1}), \quad [1+I]^{-2} = 1-2z^{-1}+z^{-2}$$

$$v(n) = u(n) + e(n) - 2e(n-1) + e(n-2)$$

$$y(n) = u(n) - 2e(n-1) + e(n-2)$$

$$|y|_{\substack{max\\min}} = |u|_{\substack{max\\min}} \pm 3|e|_{max} = |u|_{\substack{max\\min}} \pm \frac{3V_{LSB}}{2}$$

Overload limits of Q : $0 - N_{level} V_{LSB}$



So the peak-to-peak amplitude of $u(n)$ for no overload is

$$u_{p-p,max} = 9V_{LSB} - 2\times\frac{3}{2}V_{LSB} = 6V_{LSB} = 6\,\frac{1.3}{8}$$

$$u_{p-p,max} = \frac{3.9}{4} = 0.975\text{ V}$$

(3.) Loop filter for MODA

$$H_A(z) = \sum_{i=1}^{N} a_i I^i \quad ; \text{ for MODB, } a_i \to b_i$$

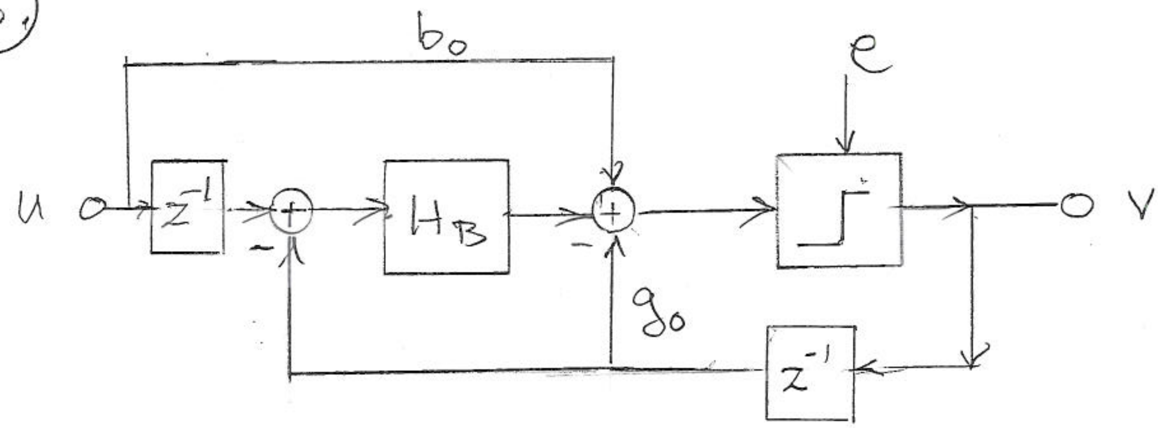
(a.) MODA

$$V = E + U + H_A(U - V)$$

$$V = U + E/(1 + H_A)$$

$$STF = 1 \,,\quad NTF = (1 + H_A)^{-1}$$

(b.)



$$V = E + b_0 U - g_0 z^{-1} V + H_B (U - V) z^{-1}$$

$$V\left[1 + (g_0 + H_B) z^{-1}\right] = E + (b_0 + z^{-1} H_B) U$$

$$NTF_B = \left[1 + (g_0 + H_B) z^{-1}\right]^{-1} \stackrel{!}{=} (1 + H_A)^{-1}$$

$$g_0 + \sum_{i=1}^{N} b_i I^i \stackrel{!}{=} z \sum_{i=1}^{N} a_i I^i$$

$$I = \frac{z^{-1}}{1 - z^{-1}} \;\longrightarrow\; z = 1 + 1/I$$

$$g_0 + \sum_{i=1}^{N} b_i I^i = (1 + 1/I) \sum_{i=1}^{N} a_i I^i$$

$$g_0 = a_1$$

$$b_i = a_i + a_{i+1} \quad , \; i = 1, 2, \ldots, N-1$$

$$b_N = a_N$$

(c.) $$STF_B = \frac{b_0 + H_B z^{-1}}{1 + (g_0 + H_B) z^{-1}} = \frac{b_0 + H_B z^{-1}}{1 + H_A}$$

$$z^{-1} H_B = H_A - g_0 z^{-1} = H_A - a_1 z^{-1}$$

$$STF_B = \frac{b_0 - a_1 z^{-1} + H_A}{1 + H_A}$$

(d.) To make $STF_B = STF_A = 1$, replace b_0 by an $F(z)$ such that

$$F(z) - a_1 z^{-1} = 1$$

$$F(z) = 1 + a_1 z^{-1}$$